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SOCIETY AFFAIRS

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

THE HYDRAULIC JUMP IN TERMS OF DYNAMIC SIMILARITY

By Boris A. Bakhmeteff,[1] M. Am. Soc. C. E., and Arthur E. Matzke,[2] Jun. Am. Soc. C. E.

Synopsis

Since Bidone's classical "Memoire"[3], the first to describe the hydraulic jump, this fascinating and puzzling phenomenon has been the subject of repeated experimental investigation. Most of the work has dealt with what may be termed the "vertical" elements of the jump, such as the relation between the lower and upper stages, the height of the standing wave, etc. Scarcely any data are available, at least in systematic form, with regard to what may be termed the "longitudinal elements", such as the length of the jump, the profile of the surface of the roller, etc. The importance of such data is obvious. In designing stilling-basins at the toes of spillways, in laying out devices to prevent erosion below sluices, and in other similar cases, knowledge of the longitudinal elements is indispensable.

During 1932–33 the longitudinal elements of the jump was the subject of systematic research in the Fluid Mechanics Laboratory of Columbia University, in New York, N. Y. A particular feature of the work was, that in interpreting and systematizing the results obtained, recourse was taken to the principle of dynamic similarity and the final data were presented in generalized dimensionless form. Dimensionless presentation in terms of dynamic similarity is known to have yielded splendid results and has become a matter of course when dealing with flow in closed conduits. On the other hand, its application to open flow, with the exception of models of rivers, has not been as widespread as could be expected. It also appears that the basic premises which should govern the approach to open-flow problems, are

Note.—Discussion on this paper will be closed in May, 1935, *Proceedings*.

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[3] *Memoires*, Acad. de Turin, 1820.

not always clearly understood and that at times investigators are prone to select parameters non-judiciously. An example is the frequent use of the so-called "Boussinesq number."

The experiments described herein were referred to a general dynamic characteristic, "the kinetic flow factor"[4]. The results obtained seem to have confirmed the usefulness of the general methods applied and are claimed to have given the first comprehensive picture of the longitudinal features of the hydraulic jump in general.

Theoretical Premises

To facilitate a comprehensive approach, a brief recapitulation will be first given of the theoretical premises, which underlie the subsequent treatment.

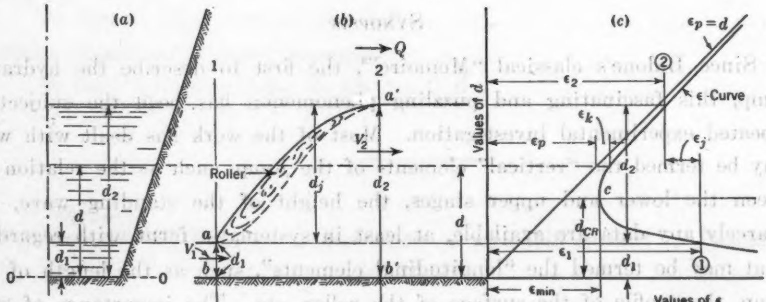

Fig. 1

Specific Energy Curve.—In treating problems of varied flow in open channels, it has become customary to refer the motion to a "specific energy curve"[5] represented in Fig. 1(*c*). The constant discharge in a canal of given cross-sectional area, A, with a varying depth, d, is expressed by Q. For each depth one may estimate the energy head (the energy in a unit weight of liquid), referred to a datum line, 0-0, drawn through the bottom of the canal. The potential energy head is $\epsilon_p = d$; the kinetic energy head is,

$$\epsilon_k = \frac{V^2}{2g} = \frac{Q^2}{2\,g\,A^2_d} \quad \text{.........................(1)}$$

in which, A_d is the cross-sectional area at the depth, d. Their sum is equal to:

$$\epsilon = \epsilon_p + \epsilon_k = d + \frac{V^2}{2g} = d + \frac{Q^2}{2\,g\,A^2_d} \quad \text{..............(2)}$$

which is the total specific energy of flow, being a function of the depth, d, only. The graph of the ϵ-curve (Fig. 1(*c*)) indicates, that the energy con-

[4] "Hydraulics of Open Channels", by Boris A. Bakhmeteff, M. Am. Soc. C. E., Engineering Society Monographs, McGraw-Hill Co., 1932, p. 64.

[5] *Loc. cit.*, p. 32.

tents vary with the depth, increasing indefinitely for very large and very small values of d. For each discharge there is a definite point, c, which marks the lowest possible energy, ϵ_{min}, compatible with the given discharge and the given canal cross-section. The flow at the point, c, is said to be "critical", the corresponding depth, d_C, being the "critical depth".

For the important case of a rectangular channel, with a width, B, and with $q = \frac{Q}{B}$, the discharge per unit width, the specific energy is:

$$\epsilon = d + \frac{q^2}{2\,g\,d^2} \qquad (3)$$

and the critical depth at which the energy content is minimum is expressed by,

$$d_C = \sqrt[3]{\frac{q^2}{g}} \qquad (4)$$

***The Hydraulic Jump.*—Referred to the specific energy curve, the hydraulic jump is a local phenomenon by means of which flow passes in a rather abrupt manner from a lower stage, d_1, corresponding to Point (1) on the lower branch of the energy curve, to the upper stage, d_2, marked by Point (2) on the upper branch (see Fig. 1(b)). Obviously, the essence of the phenomenon is the change in the form of energy. At the lower stage, energy in kinetic form prevails; in the jump, kinetic energy is transformed into potential energy, which obviously predominates on the upper branch of the curve. On the energy diagram (Fig. 1(c)), Points (2) and (1) which correspond to the so-called conjugated depths before and after the jump, do not lie on the same vertical. Their horizontal distance, $\epsilon_j = \epsilon_1 - \epsilon_2$, represents the energy head lost in the jump.**

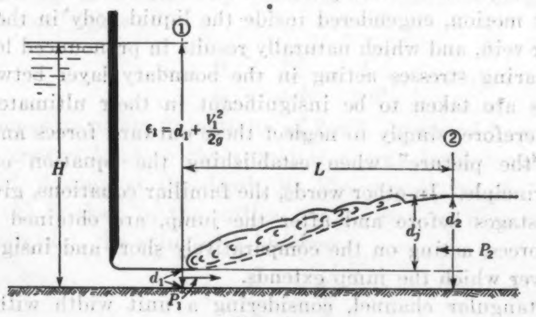

FIG. 2

Similar to many other cases of fluid dynamics in which one deals with abrupt variations in the forms of flow, a comprehensive solution is reached by applying (as first suggested by Bélanger) the "momentum principle".

It is customary to make certain fundamental assumptions when applying the momentum principle. These assumptions underlie the familiar equations relating to the hydraulic jump and should be kept clearly in mind.

In Fig. 1(b), or in Fig. 2, Sections (1) and (2) refer to the "beginning" and to the "end" of the jump. Thus, they delimit the jump, schematically,

separating it from the adjoining reaches of gradually varied flow. Motion at Section (1), preceding the toe of the roller, is free and still is not influenced by the roller. The stream lines are parallel. Therefore, the pressure distribution is assumed to follow the hydrostatic law. This means that the potential energy referred to 0-0 (Fig. 1), throughout the stream is the same and is equal to the depth, *d*, which permits expressing the average energy by the simple expression, Equation (2). Section (2) represents the "end" of the jump. It is supposed to be selected so that the expansion of the live jet under the roller has ceased. In other words, the curvature of the stream lines, which features the flow in the jump proper, is no longer present and the stream filaments have again become parallel, with the hydrostatic distribution of pressure again restored. For this reason the energy may again be computed by the simple expression, Equation (2).

The assumption of parallel filaments of flow in Sections (1) and (2) which leads to hydrostatic distribution of pressure, permits one to estimate the pressure components across these sections, which effectuate the change of momentum in the liquid body as the respective resultants of the hydrostatic pressure,

$$P = \gamma \, A_d \, z_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which, γ is the specific weight of the fluid, and z_0, the depth of the center of gravity of the respective section below the free surface.

For a jump in a horizontal flume, the only other force component acting in the direction of the flow (and thus contributing to the change of momentum) is the resultant of "external" friction forces, acting between the liquid body, *a a′ b′ b* (Fig. 1(*b*)), and the solid boundaries (the bottom and the walls) of the canal. Compared to the action of the highly disturbed and turbulent motion, engendered inside the liquid body in the roller and in the expanding vein, and which naturally results in pronounced losses of energy head, the shearing stresses acting in the boundary layer between the jump and the walls are taken to be insignificant in their ultimate effect. It is customary, therefore, simply to neglect these outward forces and to eliminate them from "the picture" when establishing the equation embodying the momentum principle. In other words, the familiar equations, giving the ratios between the stages before and after the jump, are obtained by neglecting the friction forces, acting on the comparatively short and insignificant reach, *L* (Fig. 2), over which the jump extends.

For a rectangular channel, considering a unit width with a discharge, *q*, the difference of the hydrostatic pressure is:

$$P_1 - P_2 = \gamma \left(\frac{d_1^2}{2} - \frac{d_2^2}{2} \right) \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The change of momentum per second, with the mass flow, $\frac{q\,\gamma}{g}$, is $\frac{q\,\gamma}{g}\,(V_2 - V_1)$. Equating and transforming, one obtains:

$$\frac{2\,q^2}{g} = d_1 \, d_2 \, (d_1 + d_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

from which are developed the well-known equations, giving the relations between the conjugated depths:

$$d_2 = \frac{d_1}{2}\left[-1 + \sqrt{1 + \frac{8\,q^2}{g\,d^3_1}}\right] \ldots\ldots\ldots\ldots\ldots (8)$$

and,

$$d_1 = \frac{d_2}{2}\left[-1 + \sqrt{1 + \frac{8\,q^2}{g\,d^3_2}}\right] \ldots\ldots\ldots\ldots\ldots (9)$$

Another form is obtained by substituting d_C from Equation (4), thus:

$$d_2 = \frac{d_1}{2}\left[-1 + \sqrt{1 + \frac{8\,d^3_C}{d^3_1}}\right] \ldots\ldots\ldots\ldots\ldots (10)$$

and,

$$d_1 = \frac{d_2}{2}\left[-1 + \sqrt{1 + \frac{8\,d^3_C}{d^3_2}}\right] \ldots\ldots\ldots\ldots\ldots (11)$$

Dynamic Similarity.—The Kinetic Flow Factor.—In Equations (10) and (11) the relation between the stages is made to depend on the ratio of the critical depth to the respective depths, d_2 and d_1. With reference to Fig. 1(*c*), the ratio, $\frac{d_C}{d}$, marks the position of the flow on the energy curve. The larger the ratio, $\frac{d_C}{d}$, the greater will be the velocity of flow and, consequently, the relative part of the kinetic energy component in the total energy head. In fact, each point on the energy curve characterizes a "state of flow", which may be numerically qualified by what has been termed,[4] the "kinetic flow factor"; thus:

$$\lambda = 2\,\frac{\epsilon_k}{\epsilon_p} = \frac{V^2}{g\,d} = \frac{q^2}{g\,d^3} \ldots\ldots\ldots\ldots\ldots (12)$$

which gives a measure of the "kineticity of flow", expressed by twice the ratio of the kinetic energy head to the potential energy head contained in each pound of liquid, flowing at the depth, *d*. The reason for the coefficient, 2, in Equation (12) is that, in the critical state (that is, when the flow is at the critical depth, d_C), the value of λ becomes:

$$\lambda_C = \frac{\frac{q^2}{g}}{d^3_C} = \frac{\frac{q^2}{g}}{\frac{q^2}{g}} = 1 \ldots\ldots\ldots\ldots\ldots (13)$$

In other words, the kinetic flow factor at the critical depth is $\lambda = 1$. Flow on the upper branch of the ϵ-curve, with $d > d_C$, is governed by the condition, $\lambda < 1$; flow on the lower branch, with $d < d_C$, is governed by the condition, $\lambda > 1$. Obviously, the factor, λ, is a general dimensionless characteristic of the dynamic conditions of flow. In fact, in the terms ordinarily used in studies of dynamic similarity the kinetic flow factor, λ, is equivalent to the so-called Froude number.

It is important, in this instance, to make as clear as possible the premises that underlie the treatment of open-flow problems in terms of dynamic similarity. Two basic cases must be distinguished: First, that in which the most important agencies are resistances of the frictional type, analogous to those occurring in a pipe; and, second, that in which the features of the phenomenon are, for the most part, determined by the action of gravity. The phenomena in the first case were first studied comprehensively by Osborne Reynolds.[6] Flow in these circumstances will be dynamically similar, when the same ratio persists between the inertia effects (mass times acceleration, in the Newtonian equation of motion) and the frictional resistances, the source of the origin of which is the general property of matter known as viscosity. In this case the factor expressing, numerically, the ratio of the inertia forces to the viscous forces, has the form of the so-called Reynolds number:

$$\mathsf{R} = \frac{V\,l}{\nu} \qquad (14)$$

in which, $\nu = \frac{\mu}{\rho}$, the kinematic viscosity, and l, an appropriate longitudinal parameter, such as the diameter or the radius of a closed conduit. A number similar in structure to Equation (14) would be in order, for example, when studying the frictional resistances in uniform canal flow. In that case, l could be the depth, or any other characteristic geometric dimension, provided the cross-sections compared were geometrically similar.

In the second case, the phenomena are principally determined by action of gravity. This was the problem faced by Froude, when seeking to compare the resistances offered by surface waves created by the motion of a vessel. Dynamic similarity, in this case, representing a constant ratio between inertia forces and gravity action, is expressed by a factor known as the Froude number, and usually given in the form:

$$\mathsf{F} = \frac{V}{\sqrt{g\,l}} \qquad (15)$$

in which, l is an appropriate longitudinal factor, such as the length of the ship.

In analyzing open-flow cases it must be first remembered, that the principle in general may be applied to geometrically similar cases only. Many mistakes are known to have arisen from a failure to observe this fundamental rule.

Furthermore, one must determine which of the two types of forces, gravity or friction, are preponderant in each specific case and which of the two, therefore, should be accepted as the principal agent, affecting the flow. As is well known, one cannot "cater" to both types of forces simultaneously and, therefore, a definite choice must be made.

Finally, it must be borne in mind, that the longitudinal parameter, l, in Equation (14) or Equation (15), cannot be selected at random. In fact, the parameter must correspond to the essential features of the particular case; it must be connected inherently with the physical agencies that influence one or

[6] *Philosophical Transactions*, Royal Soc., 1883.

the other of the two cases. For example, in the case of the hydraulic jump, the friction forces are neglected deliberately. The phenomenon of the "jump" is considered to depend entirely on the action of gravity, which expresses itself in the hydrostatic pressures and causes the change of momentum.

Thus, a dynamic characteristic of the Froude type is in order. A longitudinal parameter, germane to the physical essence of the phenomenon, which embodies transformation of the form of energy, is the depth, *d*, representing the potential energy of the flow. Hence, the Froude number, $\mathsf{F} = \frac{V}{\sqrt{g\,d}}$, which is equivalent to the kinetic flow factor, $\lambda = \frac{V^2}{g\,d}$.

If $\lambda = \frac{V^2}{g\,d}$ is an appropriate dynamic characteristic of the Froude type, it would naturally follow that flow in the two cases, possessing similar values of λ, would assume similar forms. Thus, for example, the ratio of the conjugated depths, $\frac{d_2}{d_1}$, or, respectively, $\frac{d_1}{d_2}$, in a hydraulic jump (see Equations (10) and (11)) may be expressed in terms of the kinetic flow factor, by substituting Equation (12) in Equations (8) and (9) which gives,

$$\frac{d_2}{d_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8\,\lambda_1}\right] \qquad (16)$$

and,

$$\frac{d_1}{d_2} = \frac{1}{2}\left[-1 + \sqrt{1 + 8\,\lambda_2}\right] \qquad (17)$$

in which, λ_1 and λ_2 are the respective kinetic flow factors in the sections before and after the jump.

Referring to the so-called Boussinesq number which, as indicated previously, is being used improperly to an ever-increasing extent:

$$\mathsf{B} = \frac{V}{\sqrt{g\,r'}} \qquad (18)$$

is an expression of the "Froude type" (Equation (15)), in which, $r' = \frac{2\,A}{\text{wetted perimeter}}$, is twice the value of Chezy's hydraulic radius. The hydraulic radius is related inherently with friction effect. It would be quite appropriate as a factor in a characteristic of the Reynolds type (Equation (14)) as, for example, in a study of friction forces in uniform flow; but it is entirely out of place in a case (such as the "jump", or flow through a structure) in which the motion is primarily dependent on gravity action. Gravity has no relation whatever to the hydraulic radius, the latter being thus a parameter that bears no physical relation to the essence of the case.

Dimensionless Representation.—Equations (16) and (17) are given in terms of ratios; that is, in a dimensionless form, which is becoming common practice in engineering science. Dimensionless representation results in equa-

tions of the most general type. Equations (16) and 17, for example, do not refer to any particular jump. They apply equally to a jump at the foot of the Boulder Dam and a small-scale model in a laboratory flume. The ratio of the depths will be the same when, and if, the kinetic flow factor is identical.

Assume, for example, that the depth of flow in any structure is d_p, with a unit-width discharge, q_p. Assume, furthermore, that the model is to be reproduced on a geometrical scale of 1:10 so that the depth of flow is $d_m = \frac{d_p}{10}$. The discharge in the model, reproducing dynamically similar conditions, requires identity of the Froude number, or an identical value of λ. Thus,

$$\frac{q^2_m}{d^3_m} = \frac{q^2_p}{d^3_p} \quad \text{.........................(19)}$$

or, with a scale of 1:10:

$$q_m = q_p \sqrt{\frac{1}{1\,000}} = 0.316\, q_p$$

General Dimensionless Characteristics of the Jump in Terms of λ.—The different features of the jump may be expressed in a useful form as ratios, $d'_1 = \frac{d_1}{\epsilon_1}$; $d'_2 = \frac{d_2}{\epsilon_1}$; etc. The physical meaning is made clear by referring to Fig. 2, in which, ϵ_1 is equal to the head, H, provided the outflow losses are omitted.

Referring to Fig. 2: $d'_1 = \frac{d_1}{\epsilon_1}$; $d'_2 = \frac{d_2}{\epsilon_1}$; $d'_j = d'_2 - d'_1$; $V^2_1 = 2\,g\,(\epsilon_1 - d_1)$; and, therefore,

$$\frac{(V'_1)^2}{2\,g} = 1 - d'_1 \quad \text{.........................(20)}$$

Furthermore, $\lambda_1 = \frac{2\,(1 - d)'_1}{d'_1}$; and,

$$d'_1 = \frac{2}{2 + \lambda_1} \quad \text{.........................(21)}$$

By means of Equation (21) and bearing in mind that $(u'_2) = u'_1 \left(\frac{d'_1}{d'_2}\right)$ one can also determine the energy in Section (2) after the jump,

$$\epsilon'_2 = d'_2 + \frac{(V'_2)^2}{2\,g} \quad \text{.........................(22)}$$

and the loss in the jump is,

$$\epsilon_j' = 1 - \epsilon'_2 \quad \text{.........................(23)}$$

The different elements of the jump may be traced[7] in terms of d'_1, or they

[7] "Hydraulics of Open Channels", Eng. Societies Monograph, 1932, p. 248.

may be presented directly in terms of the kinetic flow factor resulting in the curves of Fig. 3.

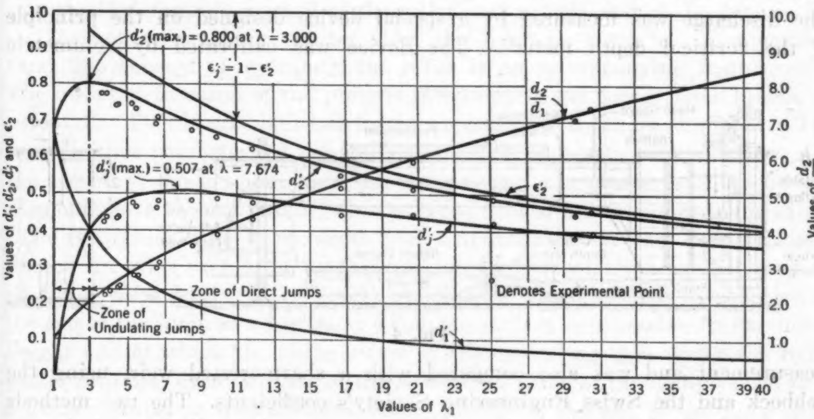

FIG. 3

These curves represent the most general dimensionless characteristics of the jump in terms of dynamic similarity. The following special features of the curves as drawn are to be emphasized as they bear directly on the experiments to follow: (1) The highest possible theoretical stage, $d'_2 = 0.800$, is reached at $\lambda_1 = 3$, the corresponding value of the initial depth being $d'_1 = 0.4$; and, (2) the d'_j-curve shows that the jump attains its maximum height at $\lambda_1 = 7.674$, with $d'_j = 0.507$.

It should be emphasized that the characteristics as given in Fig. 3 result from theoretical relations obtained by using the momentum principle. They apply to a jump in a horizontal flume, the slight effect of friction forces on the outward boundaries being neglected. It remains for experiments to show, in general, whether the foregoing premises, which permit a simple theoretical approach, are justified. The curves in Fig. 3 are supported by results obtained from experiments described subsequently. On the whole, theory and observation agree most satisfactorily. As one should expect the actual observed values of d_2 and d_j are somewhat lower than the theoretical values. This is due to the neglected effect of external friction. In general, experiments in flumes of larger size and on a larger scale, in all probability, would make the deviations still smaller.

EXPERIMENTAL PROCEDURE

The Flume.—The experiments supporting this paper were run in the varied-flow flume of the Fluid Mechanics Laboratory, at Columbia University. The tilting device, a general outline of which is given in Fig. 4, was 20 ft long, 6 in. wide, and 22 in. high; it was made of welded metal. Water was fed to the supply basin, by a 5-in. vertical pump, the basin being provided with suitable baffles and a regulated overflow.

Due to limited space and to peculiarities of the foundations the sump was of rather restricted volume, built into the lower part of the supply basin with a return channel substantially at the same level, placed under the flume. The discharge was measured by a special device designed on the principle of the "critical depth meter". The device was calibrated by volumetric

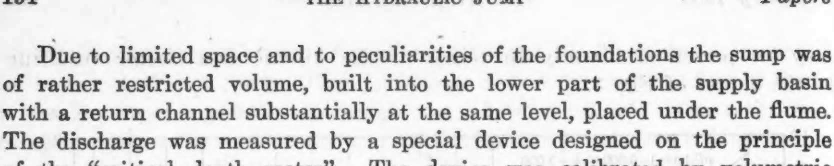

FIG. 4

measurement and was also compared with a sharp-crested weir, using the Rehbock and the Swiss Engineering Society's coefficients. The two methods gave practically identical results. It is assumed that the discharge is known accurately to within 1 per cent.

In the particular experiments referred to in this paper, the bottom of the flume was kept horizontal, and the "rapid" flow necessary for the formation of a jump, was produced by an adjustable sluice. The tail-water was regulated, and thus the location of the jump was controlled by means of an adjustable plate weir at the end of the flume. A profilometer permitted readings that were accurate to within 0.001 ft. A view of the jump "in action" has been presented elsewhere[8].

A few words of explanation may be useful to clarify some of the problems connected with measuring the hydraulic jump. The jump appears to the eye as an erratic phenomenon subject to violent and apparently inordinate oscillations. Thus, the position of the toe of the jump varies continuously, up stream and down stream. Even greater agitation is noticeable in the roller to which, at first, one may be led to doubt whether any uniform or stable laws apply. However, it is known that irrespective of the apparent convulsions, the basic dynamic relations applying to the jump (such as the ratio between the depth, d_1 and d_2) are unexpectedly uniform and constant when taken as average values over a given period. Moreover, no matter how violently the position of the jump appears to vary, a stable and definite average reading of a vertical element may be obtained over a period when, for example, the position of the pointer is adjusted so that the flow oscillates to one side and then the other, around the selected position, at more or less equal time intervals. It is in this sense that an average profile of the roller covering the expanding "live vein" of the jump may be obtained by successive vertical readings at chosen points.

Further explanation is necessary with regard to the term, "length of the jump". The beginning of the jump, at the toe of the roller, is well defined;

[8] *Civil Engineering*, November, 1934, p. 564.

but there has been some confusion as to what constitutes the "end of the jump". Some writers[9] assume the end to be substantially at a point where adverse flow is no longer observable. In the writers' experience, this definition would determine the end of the roller, and not the end of the jump.

The roller is usually shorter than the jump as a whole. Furthermore (and this concept is essential), the roller is an accompanying feature only. The essential function of the jump is to transform the energy from kinetic to potential. This is accomplished in the expanding vein under the roller. The jump is thus characterized pre-eminently by an increasing depth of flow. On the contrary, beyond the jump, after expansion has ceased and the stream filaments have become parallel (in the sense defined herein under the heading, "The Hydraulic Jump"), the depth begins to diminish—at least in a horizontal flume. In other words, in this region, flow is characterized by the lowering of the surface level. It is logical under such circumstances to conceive of the end of the jump as a section at which the surface level reaches its maximum height and at which the rising curve of the expanding vein under the roller passes into the "drop" surface of the subsequent reach of gradually varied flow. In other words, as shown schematically in Fig. 2, the end of the jump is at

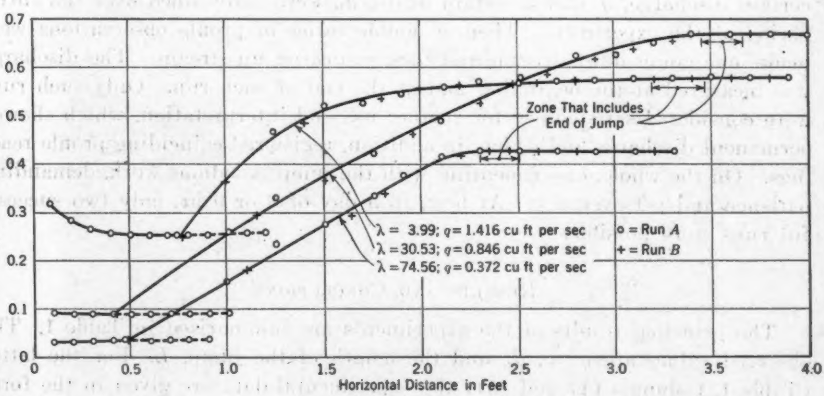

FIG. 5

Section (2) where the depth, d_2, has reached its maximum value. In practical experimental work the difficulty lies in the fact that the entire surface curve in this region is very flat and subject to continuous oscillations. In most cases, therefore, it is rather impossible to determine the position of the end section by direct observation. The method used was to trace profiles based on average observations, and then to select the end point from the drawing thus obtained. It was necessary to trace such profiles for each run (see Fig. 5). Even then, due to the flatness of the curves, all one can do is to establish a zone within which the position of the end section is more or less arbitrary. Obviously, in this instance, as in so many other cases of experimental engineering, one is dealing with a "transition zone".

[9] See, for example, *Civil Engineering*, May, 1934.

Another delicate operation is the measurement of the initial depth at the lower stage in Section (1), (Fig. 1(*a*)). The higher the kineticity, the smaller will be the value of d_1. In Equation (12), the depth, d_1, appears as a third power. Any experimental error, therefore, affects the final result seriously. These facts define a natural limit to experimentation. One cannot increase λ_1 by decreasing d_1 beyond a reasonable amount. Then (see Fig. 2), the initial depth must be measured at the toe of the roll, which is in an oscillating state and which obviously interferes with the pointer of the profilometer. The procedure used was again to trace an auxiliary profile of the free vein over a certain stretch by moving the jump somewhat downward. After the profile was thus determined, the jump was brought back into position, and the respective depth, d_1, was taken from the auxiliary tracing. The procedure is clearly demonstrated in Fig. 5.

The actual experiments were run during a period of about ten months, beginning December, 1932. At least one-half the time was spent on calibration and preliminary work, in which methods of procedure were gradually developed. Most of these preliminary results were discarded and, with the experience and skill gained, a second and final set was run. For each run a certain discharge, q, and a certain depth, d_1, were maintained over the entire period of the experiment. Then, a double series of profile observations were made, one going down stream and then returning up stream. The discharge was measured at the beginning and at the end of each run. Only such runs were considered satisfactory for further use and interpretation, which showed permanent discharge and which, in addition, registered coinciding profile readings. On the whole, experimenting with the jump is tedious work, demanding patience and perseverance. At best, in a day of 8 or 9 hr, only two successful runs were possible.

Results and Conclusions

The principal results of the experiments are summarized in Table 1. The observed values are q, d_1, d_2, and the length of the jump, L. For the latter (Table 1, Columns (4) and (5)) the experimental data are given in the form of a range as shown by Fig. 5. The other tabular values were computed. The basic factor is, $\lambda_1 = \frac{q^2}{g\, d^3_1}$; and the initial energy, $\epsilon_1 = d_1 + \frac{q^2}{2g\, d^2_1}$. The ratio, $\frac{d_2}{d_1}$, in Column (12), Table 1, is the quotient, and d_j in Column (7) is the difference of the values in Columns (3) and (2). The dimensionless factors in Columns (9), (10), and (11), Table 1, are obtained by dividing Columns (2), (3), and (7) by Column (8).

Length of the Jump.—The first and immediate aim of the research undertaken was to obtain comprehensive data concerning the length of the jump. Columns (13) to (20), Table 1, give the length in terms of dimensionless ratios, the observed L-values, Columns (4) and (5), being divided by Columns (2), (3), (7), and (8). The results are represented in Fig. 6. In tracing

TABLE 1.—Observations on the Hydraulic Jump; Experiments at Columbia University

Run No.	Unit discharge, q, in cubic feet per second	Depth of Flow d, in Feet (See Fig. 2): Section (1)	Section (2)	Length of Jump L, in Feet, Ranging: From	To	Kinetic flow factor, λ_1	Height of jump, d_j, in feet	Energy head, ϵ_1, in feet	Ratios d' at Sections: $d'_1 \left(= \frac{d_1}{\epsilon_1}\right)$	$d'_2 \left(= \frac{d_2}{\epsilon_1}\right)$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
S 27	1.400	0.253	0.561	2.30	2.70	3.76	0.308	0.729	0.347	0.769
S 30	1.416	0.251	0.574	2.44	2.84	3.94	0.323	0.746	0.337	0.769
S 40	1.286	0.225	0.541	2.23	2.63	4.51	0.316	0.733	0.307	0.739
S 43	1.312	0.228	0.550	2.47	2.87	4.52	0.322	0.743	0.307	0.740
S 25	1.680	0.254	0.693	2.86	3.36	5.35	0.439	0.934	0.272	0.742
S 45	1.648	0.249	0.681	2.86	3.26	5.47	0.432	0.930	0.268	0.732
S 41	1.590	0.228	0.681	3.23	3.53	6.63	0.453	0.984	0.232	0.692
S 24	1.816	0.248	0.765	3.65	4.05	6.72	0.517	1.081	0.229	0.708
S 28	2.060	0.249	0.881	4.19	4.49	8.55	0.632	1.313	0.190	0.671
S 26	2.284	0.254	0.989	4.87	5.17	9.90	0.735	1.511	0.168	0.654
S 29	2.030	0.221	0.957	4.95	5.25	11.87	0.736	1.533	0.144	0.624
S 36	1.600	0.168	0.869	4.35	4.65	16.79	0.701	1.578	0.107	0.551
S 18	1.148	0.125	0.742	3.60	3.90	20.75	0.617	1.397	0.091	0.515
S 6	0.988	0.107	0.705	3.23	3.53	25.10	0.598	1.443	0.074	0.488
S 17	0.820	0.089	0.630	2.94	3.24	29.66	0.541	1.409	0.063	0.447
S 39	0.846	0.090	0.663	3.05	3.25	30.53	0.573	1.464	0.062	0.453
S 35	0.586	0.062	0.536	2.42	2.62	44.81	0.474	1.451	0.043	0.369
S 37	0.508	0.053	0.503	2.24	2.44	53.89	0.450	1.481	0.036	0.340
S 32	0.456	0.047	0.477	2.01	2.21	62.25	0.430	1.510	0.031	0.316
S 34	0.376	0.040	0.435	1.90	2.00	68.70	0.395	1.414	0.028	0.308
S 33	0.372	0.039	0.428	1.81	2.01	74.56	0.389	1.454	0.027	0.294
S 38	0.288	0.032	0.379	1.47	1.77	78.69	0.347	1.291	0.025	0.294

Run No.	Ratio, $d'_j \left(= \frac{d_j}{\epsilon_1}\right)$	Ratio, $\frac{d_1}{d_2}$	Ratios of Comparison, Ranging: $\frac{L}{d_1}$ From	To	$\frac{L}{d_2}$ From	To	$\frac{L}{d_j}$ From	To	$\frac{L}{\epsilon_1}$ From	To
	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
S 27	0.423	2.217	9.09	10.67	4.10	4.81	7.47	8.77	3.15	3.70
S 30	0.433	2.287	9.72	11.31	4.25	4.95	7.55	8.79	3.27	3.81
S 40	0.432	2.406	9.91	11.69	4.12	4.86	7.06	8.32	3.04	3.59
S 43	0.433	2.412	10.83	12.59	4.49	5.22	7.67	8.91	3.32	3.86
S 25	0.471	2.730	11.26	13.23	4.13	4.85	6.51	7.65	3.06	3.60
S 45	0.463	2.733	11.49	13.09	4.20	4.79	6.62	7.55	3.07	3.51
S 41	0.460	2.987	14.17	15.48	4.74	5.18	7.13	7.79	3.28	3.59
S 24	0.479	3.087	14.72	16.33	4.77	5.29	7.06	7.83	3.38	3.75
S 28	0.481	3.538	16.83	18.03	4.75	5.10	6.63	7.10	3.19	3.42
S 26	0.486	3.892	19.17	20.35	4.92	5.23	6.63	7.03	3.22	3.42
S 29	0.480	4.330	22.40	23.75	5.17	5.49	6.73	7.13	3.23	3.42
S 36	0.444	5.173	25.89	27.68	5.01	5.35	6.21	6.63	2.76	2.95
S 18	0.441	5.913	28.80	31.20	4.85	5.26	5.83	6.32	2.58	2.79
S 6	0.415	6.615	30.19	32.99	4.58	5.01	5.40	5.90	2.24	2.45
S 17	0.384	7.079	33.03	36.40	4.67	5.14	5.43	5.99	2.09	2.30
S 39	0.391	7.372	33.89	36.11	4.60	4.90	5.32	5.67	2.08	2.22
S 35	0.327	8.645	39.03	42.26	4.51	4.89	5.11	5.53	1.67	1.81
S 37	0.304	9.490	42.26	46.04	4.45	4.85	4.98	5.42	1.51	1.68
S 32	0.285	10.159	42.77	47.02	4.21	4.63	4.67	5.14	1.33	1.46
S 34	0.279	10.888	47.50	50.00	4.37	4.60	4.81	5.06	1.34	1.41
S 33	0.267	10.974	46.41	51.54	4.23	4.70	4.65	5.17	1.24	1.38
S 38	0.269	11.860	45.94	55.31	3.88	4.67	4.24	5.10	1.14	1.37

the curves, it is obvious that they must pass within the plotted ranges, indicating the possible range of the L-values. Then, the investigator should be guided by the interrelation between d_2, d_1, d_j, etc., as illustrated by Fig. 3. In other words, when a certain region happens to be marked by particularly scattered data, the curve in question can be traced, nevertheless, by making use of some other curve, which in that particular region will be experi-

mentally defined with better precision. The upper limit of λ_1 practicable in the case, could not be extended beyond 70 to 80 (see Fig. 6). The lower limit (slightly more than 3.5) is determined, on the other hand, by certain physical features inherent in the phenomenon. As indicated previously in another connection,[10] the maximum d_2-point in Fig. 3, corresponding theoretically to $\lambda_1 = 3$, delimits two possible forms of the jump: (1) The direct jump illustrated by Fig. 5, and the undulating jump in Fig. 7. Obviously, the region near $\lambda_1 = 3$ is a zone of transition, since it is reasonable to anticipate that the

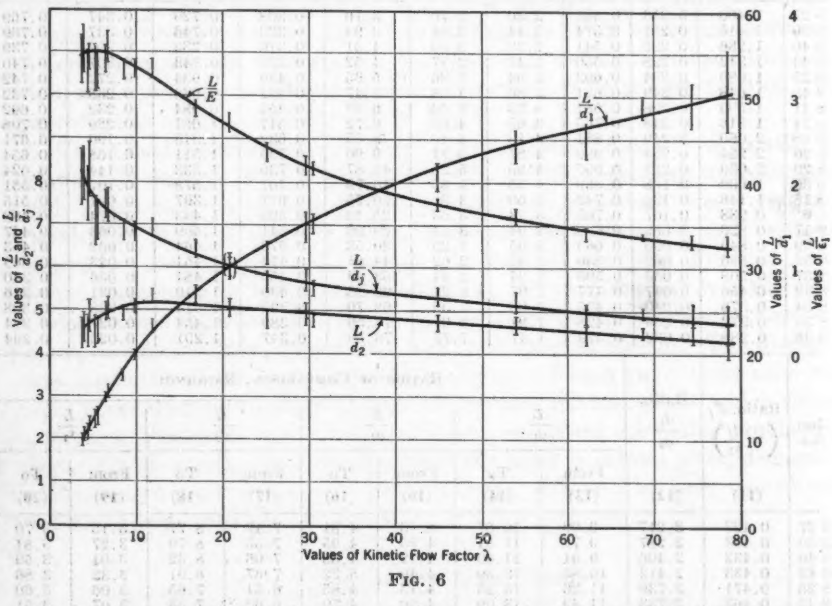

FIG. 6

change to the undulating form is gradual and that the undulations increase as λ_1 is diminished. Fig. 7 gives a striking confirmation of the foregoing statements. The discharge being kept constant, the sluice was gradually opened and the kineticity decreased. The undulations become more and more pronounced. No numerical values for λ are given in the diagram, because the vein at the lower stage ceased to be parallel. Due to curvature the distribution of pressure is no longer hydrostatic and the simple expression (Equation (2)) for ε is no longer applicable. One may gather from Fig. 3 that the effect of boundary friction (disregarded in the Bélanger theory) would be to shift the actual point, d'_2 (maximum), somewhat to the right. For this reason, the zone dividing direct and undulating jumps would be actually at a somewhat greater value than the theoretical, $\lambda = 3$; possibly $\lambda = 3.5$, or 4.0.

Generalized Profiles of the Jump.—In plotting the profiles, in Fig. 5, the writers became impressed by the fact that the outlines of apparently such an irregular and capricious phenomenon as the jump finally proved to be un-

[10] "Hydraulics of Open Channels", Eng. Societies Monograph, 1932, p. 249

expectedly regular. The idea naturally arose to reduce all the observed profiles to some unified dimensionless form. After certain trials, it appeared expedient to use the height of the jump, $d_j = d_2 - d_1$, as the parameter for reference. The different profiles were referred to a system of co-ordinates,

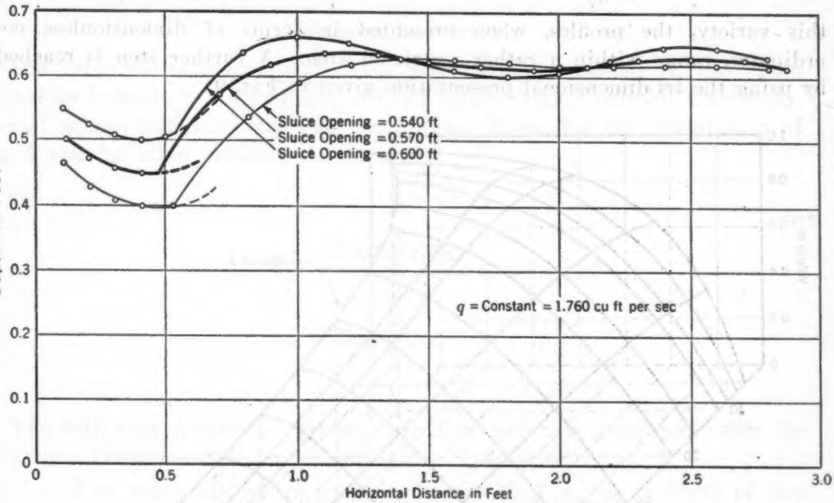

FIG. 7

shown in Fig. 8(*a*). The zero point coincides with the beginning of the jump. The co-ordinates of the surfaces are made dimensionless by introducing the ratios, $\frac{x}{d_j}$ and $\frac{y}{d_j}$. A series of profiles was traced in this manner, a selection of which is given in Fig. 8. Intermediate profiles are omitted in order to avoid obscuring the curves.

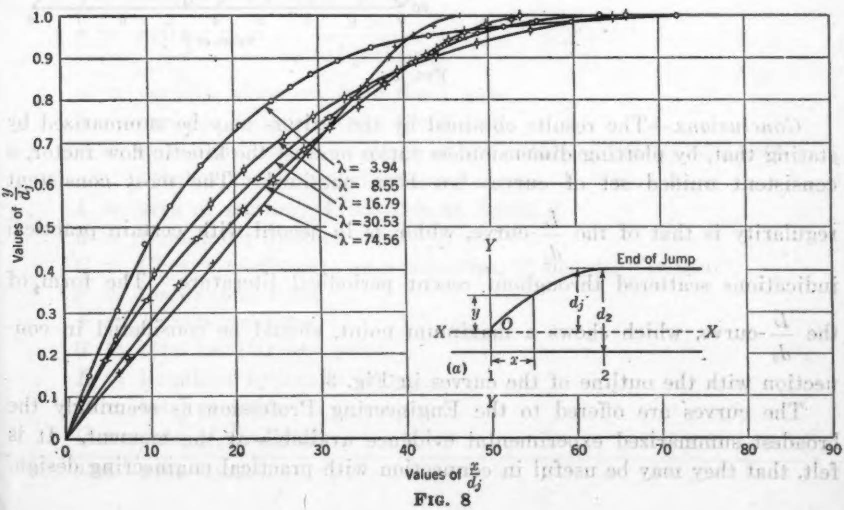

FIG. 8

The jumps embodied in these profiles are typical of a large variety. The initial depth varies from 0.03 to 0.25 ft; the length, L, from 1.5 to more than 5 ft; and the ratio, $\frac{d_2}{d_1}$, changes from about 2 to nearly 12. Notwithstanding this variety, the profiles, when presented in terms of dimensionless co-ordinates, range within a rather restricted area. A further step is reached by using the tri-dimensional presentation given in Fig. 9.

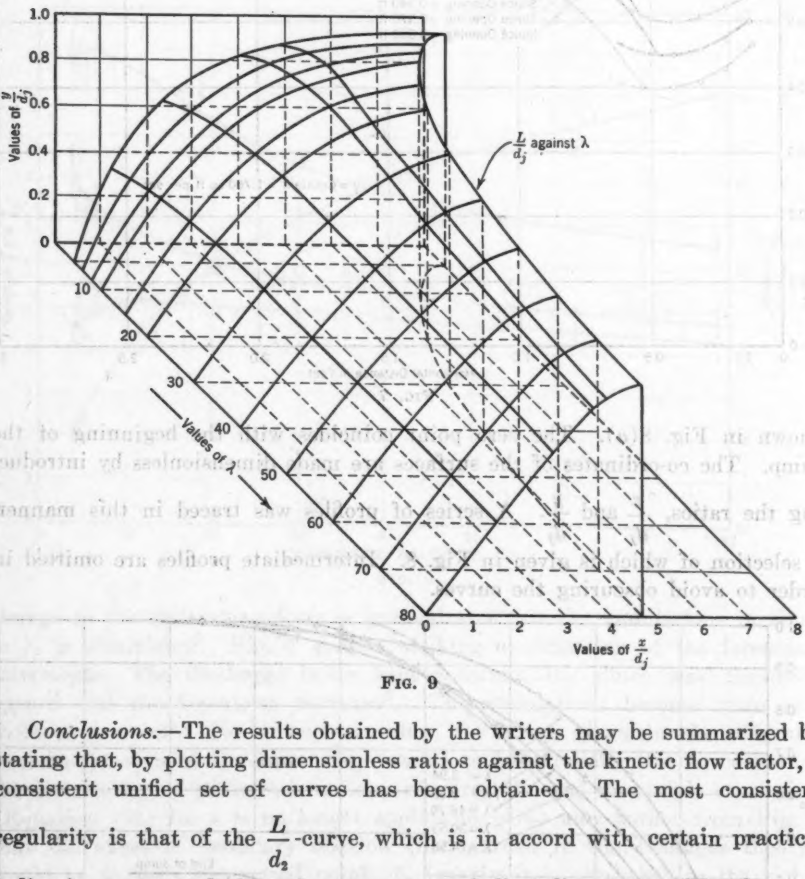

FIG. 9

Conclusions.—The results obtained by the writers may be summarized by stating that, by plotting dimensionless ratios against the kinetic flow factor, a consistent unified set of curves has been obtained. The most consistent regularity is that of the $\frac{L}{d_2}$-curve, which is in accord with certain practical indications scattered throughout recent periodical literature. The form of the $\frac{L}{d_2}$-curve, which shows a maximum point, should be considered in connection with the outline of the curves in Fig. 3.

The curves are offered to the Engineering Profession as seemingly the broadest summarized experimental evidence available at the moment. It is felt, that they may be useful in connection with practical engineering design.

The writers are fully conscious of the fact that the size of the flume and certain imperfections of its construction militate against a greater precision in the results. It is believed, however, that the general aspects of the phenomenon of the hydraulic jump have been made clearer.

Acknowledgment

The experiments described in this paper were conducted in the Fluid Mechanics Laboratory, Department of Civil Engineering, Columbia University. Special acknowledgments are due Dr. Hunter Rouse for the preparation of Fig. 9 and for other valuable assistance.

APPENDIX

Notation

The following notation, introduced in this paper, is consistent with the American Tentative Standard Symbols for Hydraulics[11]:

d = depth of flow = potential energy head, ε_p; d_p = depth of flow in the prototype; d_m = depth of flow in a model.
g = acceleration due to gravity.
j = a subscript denoting "hydraulic jump".
k = a subscript denoting "kinetic"
l = an appropriate longitudinal parameter.
m = a subscript denoting "model"
p = intensity of pressure; as a subscript, p denotes "prototype".
q = discharge per unit width.
r = radius; $r' = \dfrac{2\,A}{\text{wetted perimeter}}$.
x = variable distances from the Y-axis
y = variable distances from the X-axis.
z = elevation head; z_0 = depth of the center of gravity of a section below the free surface.
A = area of section; A_d = area at depth, d.
B = Boussinesq number.
C = Chezy's coefficient; as a subscript, C, denotes "critical".
F = force; total force.
F = Froude's number.
H = total head at any point.
L = length of hydraulic jump
P = total pressure.
Q = rate of discharge.

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[11] A. S. A.—Z10b—1929.

R = hydraulic radius.
R = Reynolds number.
V = average velocity at a section.
γ = specific weight of a fluid.
ϵ = energy head, or total specific energy of flow; ϵ_p = potential energy; ϵ_k = kinetic energy, ϵ_j = energy loss in the hydraulic jump.
λ = kinetic flow factor.
μ = absolute viscosity = $\nu \rho$.
ρ = density.
ν = kinematic viscosity $= \dfrac{\mu}{\rho}$.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

FRICTIONAL RESISTANCE IN ARTIFICIALLY ROUGHENED PIPES

By Victor L. Streeter,[1] Jun. Am. Soc. C. E.

Synopsis

The flow of fluids in smooth pipes is so well understood that the losses due to friction may be predicted to within 5 per cent. The problem of estimating friction losses in old pipes, or in those roughened by use, however, has been inadequately solved. In order to clarify the problem completely much study and experimentation are necessary. This paper, which is an effort in that direction, presents the results of an experimental investigation of frictional resistance in artificially roughened pipes. It has been undertaken in an effort to show, qualitatively, the effect on the friction factor of certain artificial irregularities—varying in shape and size—that were introduced in pipes used for the tests. With water as the fluid, loss of head was investigated for roughened, 2-in., brass pipe, the Reynolds numbers ranging from 20 000 to 1 250 000. The roughness elements consisted of grooves cut spirally into the pipe. The degree of roughness was varied by changing the depth, shape, and number of grooves per inch.

The loss of head due to friction was found to increase with an increase in the depth of the grooves, provided the same general shape was maintained. The shape of the grooves, however, seems to have almost as much affect on the loss of head as the depth. The experiments also indicate that the roughness elements may be placed so close together that the loss of head is reduced. The work of previous investigators is summarized briefly; the apparatus, preliminary experiments, experiments for obtaining data, and computations are described; and the results of this investigation are discussed.

Notation

The symbols in this paper are introduced in the text as they occur and are summarized for reference in Appendix I. An effort has been made to

Note.—Discussion on this paper will be closed in May, 1935, *Proceedings*.

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conform essentially with "Symbols for Hydraulics,"[2] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1929.

PREVIOUS INVESTIGATIONS

Investigations of fluid flow in pipes have been concerned with the development of general formulas, and, more recently, with studies into the effect of artificial roughness on the friction factor.

General Considerations of Fluid Flow.—In the evolution of formulas dealing with the flow of fluid in pipes, the findings of Hagen, Poiseuille, Darcy, Reynolds, and Blasius are especially significant. In 1839, Hagen[3] observed that there were two different types of fluid flow through pipes, now commonly designated as stream-line and turbulent flow. Thirty years later he stated that the point at which transfer from one form of flow to the other occurred depended on the radius, the velocity, and the temperature of the water.

In 1845, Poiseuille[4] experimented with the flow of water through capillary tubes. He deduced the following law for stream-line flow, known as Poiseuille's law, which has since been derived theoretically:

$$F_l = \frac{64\,\mu\,L\,V}{2\,g\,D^2} \quad \text{(1)}$$

in which, F_l = loss of force, or pressure; μ = absolute viscosity; L = length of experimental pipe, in feet; V = average velocity, in feet per second; g = acceleration due to gravity; and D = mean diameter of pipe, in feet. Equation (1) may be expressed in terms of the Reynolds number and the Darcy friction factor, as follows:

$$f = \frac{64}{\mathsf{R}} \quad \text{(2)}$$

in which, f = Darcy's friction factor; and R = Reynolds number.

In 1857, Darcy[5] conducted a series of experiments on the flow of water in cast-iron pipes with diameters ranging from 0.5 in. to 20 in., obtaining the formula,

$$H_f = \frac{V^2 L}{c^2 D} \quad \text{(3)}$$

in which, H_f = loss of head due to friction; and c^2 = a coefficient which Darcy considered to be constant for each type of surface. The Darcy formula is now usually written in the form,

$$H_f = \frac{f\,L\,V^2}{2\,g\,D} \quad \text{(4)}$$

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[2] A. S. A.—*Z10b*—1929.

[3] "Über die Bewegung des Wassers in engen zylindrischen Röhren," von G. Hagen, *Poggenborff's Annalen*, Bd. 46, S. 423, 1839.

[4] "Recherches expérimentales sur le mouvement des liquides dans tubes de très petits diamètres," par Poiseuille, *Comptes Rendus*, Vol. 11, 1840, pp. 961, 1041; Vol. 12, 1841, p. 112; also, *Mémoires* des Savants Etrangers, Vol. 9, 1846.

[5] "Recherches expérimentales relatives au mouvement de l'eau dans les tuyaux," par H. Darcy, *Mémoires* des Savants Etrangers, Vol. 15, 1858, p. 141.

in which, $f = \dfrac{2\,g}{c^2}$. The friction factor, f, will be used throughout this paper as the measure of frictional resistance.

By a study of Stokes' equations of motion, Reynolds[6] conceived the idea that a criterion to determine whether flow would be stream-line or turbulent might be expressed in the form,

$$\mathsf{R} = \frac{D\,V\,\rho}{\mu} \quad \text{.......................} \quad (5)$$

in which, ρ = density, in pounds per cubic foot. In 1883 he confirmed this theory experimentally by observing dye introduced into flowing water in glass tubes. The value of Reynolds number, $\dfrac{D\ V\ \rho}{\mu}$, at which change from one flow form to the other takes place is modified more or less by conditions at the inlet of the pipe, by roughness, the amount of curvature, etc. A reasonable mean value to accept as the transition point between stream-line and turbulent flow is $\mathsf{R} = 2\,100$.

Working with the experimental data of Saph and Schoder[7] on flow of water in smooth brass pipe, Blasius[8] obtained the empirical formula,

$$f = 0.316\ \mathsf{R}^{-0.25} \quad \text{.......................} \quad (6)$$

Reynolds numbers for these experiments were less than 100 000. Later experiments on smooth pipe show that the Blasius formula does not fit the data for Reynolds numbers greater than 100 000. Nikuradse[9] obtained the relation,

$$f = 0.0032 + 0.221\ \mathsf{R}^{-0.237} \quad \text{....................} \quad (7)$$

from his experiments on smooth pipe, with R as high as 3 200 000. Drew, Koo, and McAdams[10] obtained the formula,

$$f = 0.0056 + 0.500\ \mathsf{R}^{-0.32} \quad \text{....................} \quad (8)$$

from a study of all available experimental data on smooth pipes.

Experiments with Artificially Roughened Pipes.—During the fifteen years since about 1920, Schiller, Hopf, Fromm, and Nikuradse have experimented with artificially roughened pipes to determine whether roughness follows the law of similitude. In order to have similitude, corresponding surface measurements or dimensions of roughness elements in pipes must be in the same ratio as the radii, such that, if k is the measure of absolute roughness

[6] "An Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and of the Law of Resistance in Parallel Channels," by Osborne-Reynolds, *Philosophical Transactions*, Royal Soc., Lond, 1883; or *Scientific Papers*, Vol. II, p. 51.

[7] *Transactions*, Am. Soc. C. E., Vol. LI (1903), p. 253.

[8] "Das Ähnlichkeitsgesetz bei Reigungsvorgängen," von H. Blasius, *Physicalische Zeitschrift*, Vol. 12, p. 1175, 1911; or, *Forschungsarbeiten Ingenieur*, Heft 131.

[9] "Gesetzmässigkeiten der turbulenten Strömung in glatten Röhren," von J. Nikuradse, Verein Deutscher Ingenieur, *Forschungsheft*, No. 356, 1932.

[10] "The Friction Factor for Clean Round Pipes," by Drew, Koo, and McAdams, *Journal*, Am. Inst. of Chemical Engrs., Vol. 28, p. 56, 1932.

and r is the radius of the pipe, the ratio, $\frac{k}{r}$, is a constant. Factor k, expressed in inches, is a complete measure of the size, shape, and distribution of the individual roughness elements. As there is at present no method for measuring or defining roughness, the symbol, k, is hypothetical.

Schiller[11] experimented with artificially roughened pipe of three sizes: 8 mm, 16 mm, and 21 mm, in inside diameter (1 in. = 25.3998 mm). The roughness consisted of spiral threads cut into the inner surfaces. Two arrangements of threads were utilized. In one, the pitch was 0.8 mm, and the depth, 0.6 mm; and in the other, the pitch was 0.4 mm, and the depth, 0.3 mm. The results of Schiller's tests are shown in Fig. 1. Curve 3 indi-

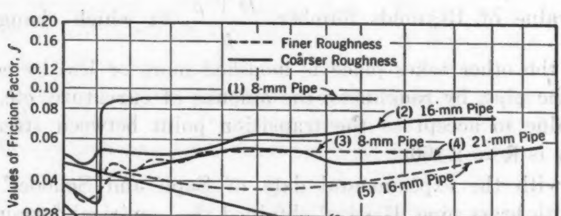

FIG. 1.—RESULTS OF EXPERIMENTS BY SCHILLER.

cates results for the 8-mm pipe with the finer roughness, and Curve 2, the 16-mm pipe with the coarser roughness. If the experiments on the two pipes conformed to the principles of similitude, Curves 3 and 2 should coincide since the roughness elements were proportional to the radii. The discrepancy may be attributed to the short calming lengths which appear to have been inadequate, and also to the fact that the roughness in the calming section of each tube differed from that in the test portion.

In investigating roughness in rectangular closed conduits, Ludwig Hopf[12] came to the conclusion that the friction factor depended upon three quantities: The cross-section of the conduit; Reynolds number; and a number which characterizes the roughness. He stated that the number for specifying the roughness appeared to require several dimensionless quantities to specify completely its size, shape, and distribution. Hopf also decided that there were two kinds of roughness, which he called "wall-waviness" and "wall-roughness." An example of the first is wood pipe or asphalted iron pipe, and an example of the second type is provided by the bare surface of cast-iron pipe.

Fromm's[13] investigations of relative roughness were carried on shortly after those of Hopf and with the same experimental equipment. In each of

[11] "Uber den Strömungswiderstand von Röhren verschiedenen Querschnitts und Rauhigkeitsgrades," von L. Schiller, *Zeitschrift angewandete Mathematik und Mechanik*, Vol. 3, pp. 2–13, 1923.

[12] "Die Messung der hydraulischen Rauhigkeit," von Ludwig Hopf, *Zeitschrift angewandete Mathematik und Mechanik*, Vol. 3, pp. 329–339, 1923.

[13] "Strömungswiderstand in rauhen Röhren," von K. Fromm, *Zeitschrift angewandete Mathematik und Mechanik*, Vol. 3, pp. 339–358, 1923.

the roughnesses tested, several values of the hydraulic radius were used. For those cases in which f becomes constant for larger values of R, the effect of the change of hydraulic radius may be expressed by the formula,

$$f = (\text{constant})\ d^{-0.314} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which, d is the depth of section, and the constant is dependent upon the absolute roughness, k', of the surface.

Whereas Hopf, Fromm, and Schiller investigated relative roughness over a comparatively limited range of Reynolds numbers, Nikuradse[14] obtained values of f within the range of turbulent flow up to a Reynolds number of 1 000 000. He used roughnesses varying from fairly smooth surfaces to projections equal to one-fifteenth of the radius. These roughnesses were produced by glueing sand to the inside of three sizes of pipes (diameters, 2.5 cm, 5 cm, and 10 cm). The diameter of the sand grain, k', was taken as the measure of absolute roughness, k, the relative roughness being $\frac{k'}{r}$. Sand was selected of such diameter that $\frac{k'}{r}$ was the same for the three different sizes of pipes. Six different values of $\frac{k'}{r}$ were used. The results of the experiments are shown in Fig. 2. The tests show that the principle of similitude applies accurately

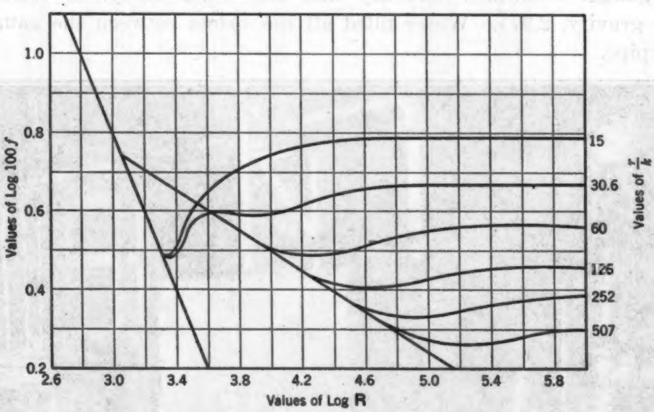

FIG. 2.—RESULTS OF EXPERIMENTS BY NIKURADSE.

to pipes roughened in this manner. For the range of Reynolds numbers in which f is constant, Nikuradse gives the formula,

$$f = \frac{1}{\left(1.74 + 2 \log_{10} \frac{r}{k'}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

[14] "Strömungsgesetze in rauhen Röhren," von J. Nikuradse, Verein Deutscher Ingenieur, *Forschungsheft*, No. 361, 1933.

Apparatus

The experiments undertaken in this investigation were conducted at the University of Michigan, Ann Arbor, Mich. The apparatus is illustrated in Figs. 3 and 4. The quantity of water used in the experiments varied from

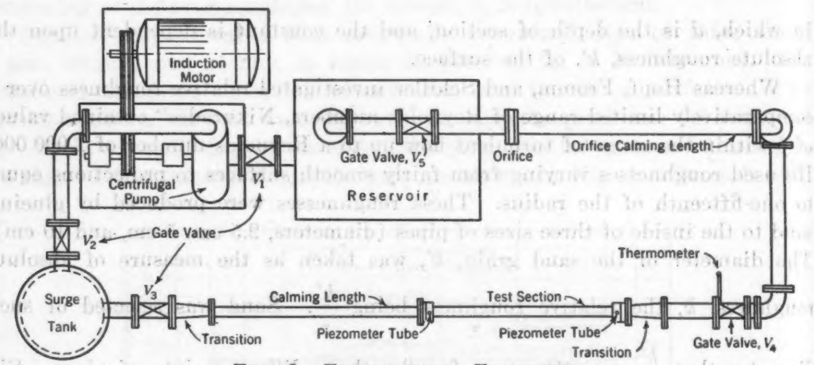

FIG. 3.—EXPERIMENTAL EQUIPMENT.

10 to 400 gal per min. Two orifices were used for measuring, one for small, the other for large, flows. Fig. 5 shows the orifice layout. Two differential manometers were used to measure the pressure drop across the orifice. One of the gauges contained mercury and the other acetylene tetra-bromide (specific gravity, 2.97). Water filled all the spaces between the gauge liquid and the pipe.

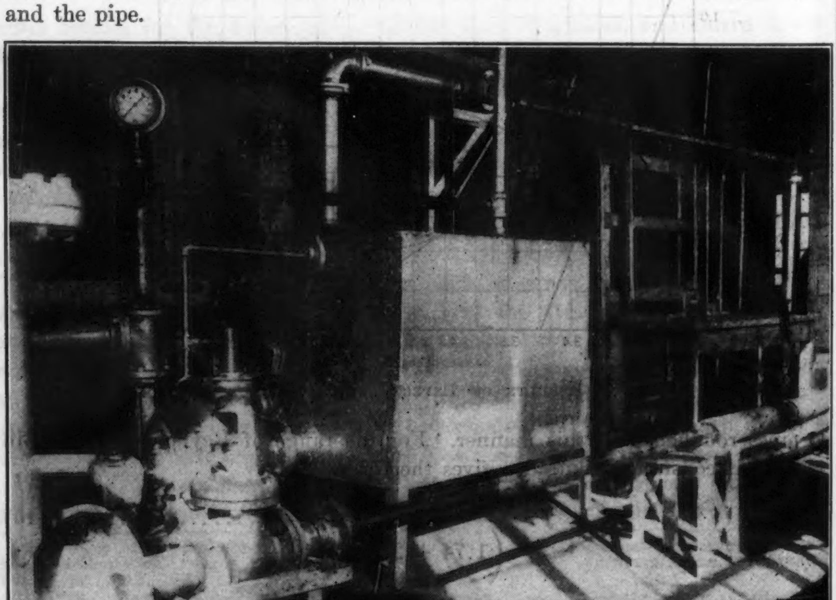

FIG. 4.—VIEW OF EXPERIMENTAL EQUIPMENT.

As the accuracy of piezometer rings is questionable in rough pipes, piezometer tubes of special design (see Fig. 6) were used to measure the pressure drop due to friction. These tubes were closed at the end, with a transverse

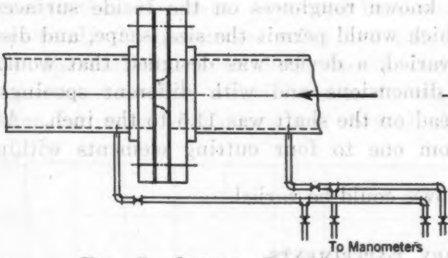

FIG. 5.—ORIFICE LAYOUT.

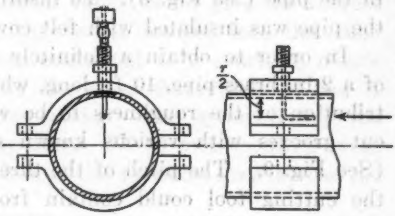

FIG. 6.—PIEZOMETER TUBE.

opening $\frac{1}{4}$ in. from the end of the tube. A diagram of the test-section manometers is shown in Fig. 7. Details of connections of copper tubing to manometer glass are illustrated in Fig. 8. Manometer *A*, Fig. 7, contained mercury; Manometer *B* contained acetylene tetra-bromide; and

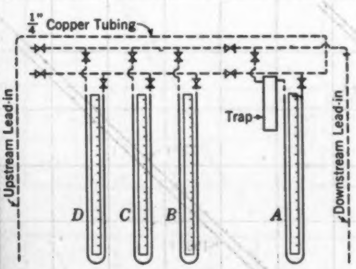

FIG. 7.—TEST SECTION, MANOMETER SYSTEM.

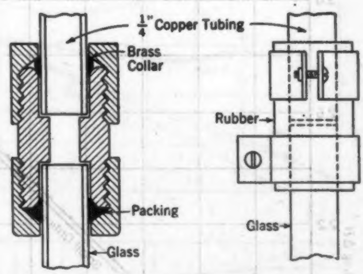

FIG. 8.—MANOMETER CONNECTIONS.

Manometers *C* and *D* contained solutions of acetylene tetra-bromide and xylene, with specific gravities, respectively, of 1.3 and 1.05. All of them were differential manometers with water acting as one liquid.

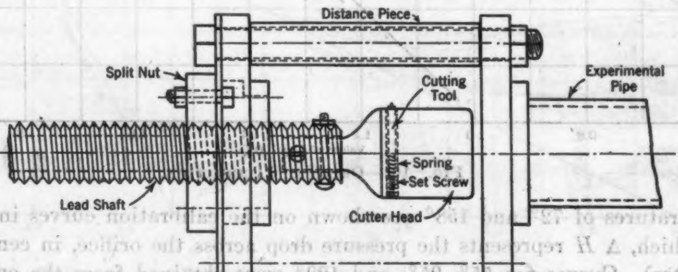

FIG. 9.—ROUGHENING APPARATUS.

A steam coil in the reservoir was used to heat the water to the desired temperature. A thermometer, reading to 110° C, was inserted in the valve at the down-stream end of the test section to obtain the temperature of water in the pipe (see Fig. 3). To insure isothermal flow through the test section, the pipe was insulated with felt covering.

In order to obtain a definitely known roughness on the inside surfaces of a 2-in. brass pipe, 10 ft. long, which would permit the size, shape, and distribution of the roughness to be varied, a device was designed that would cut grooves with various known dimensions and with different spacings. (See Fig. 9.) The pitch of the thread on the shaft was 11.5 to the inch. As the cutting tool could contain from one to four cutting elements within $\frac{1}{11.5}$ in., the distribution of the grooves could be varied.

Preliminary Experiments

The preliminary experimental work included orifice calibrations, piezometer tests, and smooth pipe tests. The orifices were calibrated over the range of flows, for temperature of 72° and 158° F. Partial calibrations were made at temperatures of 55° F and 122° F. The experimental data for

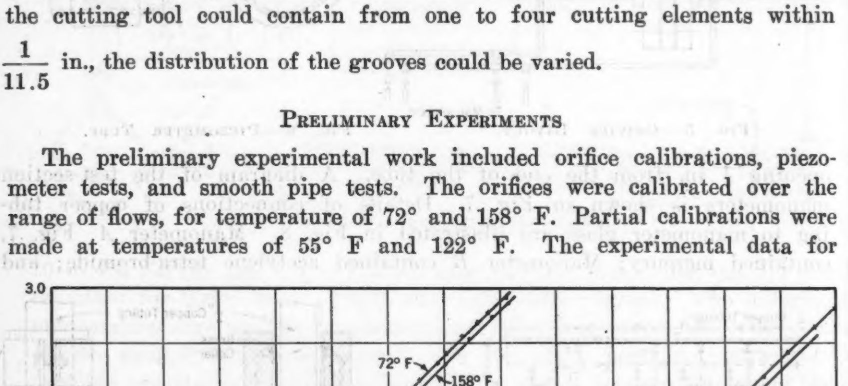

Fig. 10.—Orifice Calibration.

temperatures of 72° and 158° are shown on the calibration curves in Fig. 10 (in which, ΔH represents the pressure drop across the orifice, in centimeters of water). Curves for 55°, 95°, and 122° were obtained from the curves for 72° and 158°, by assuming the change of discharge for a given pressure drop

across the orifice to be directly proportional to the change in viscosity of the water. A change of temperature from 55° to 158° increased the discharge by as much as 6.5% for a given pressure drop.

In order to be sure that the two piezometer tubes would give the correct difference in pressure in the test section, they were inserted into the same cross-section of a 2-in. pipe, and both were connected to a sensitive manometer. If the piezometer tubes had exactly the same characteristics and if the distribution of velocities in the cross-section were symmetrical, the manometer reading would be zero for any flow. These test readings were taken for several velocities, and they all showed the pressure drop across the

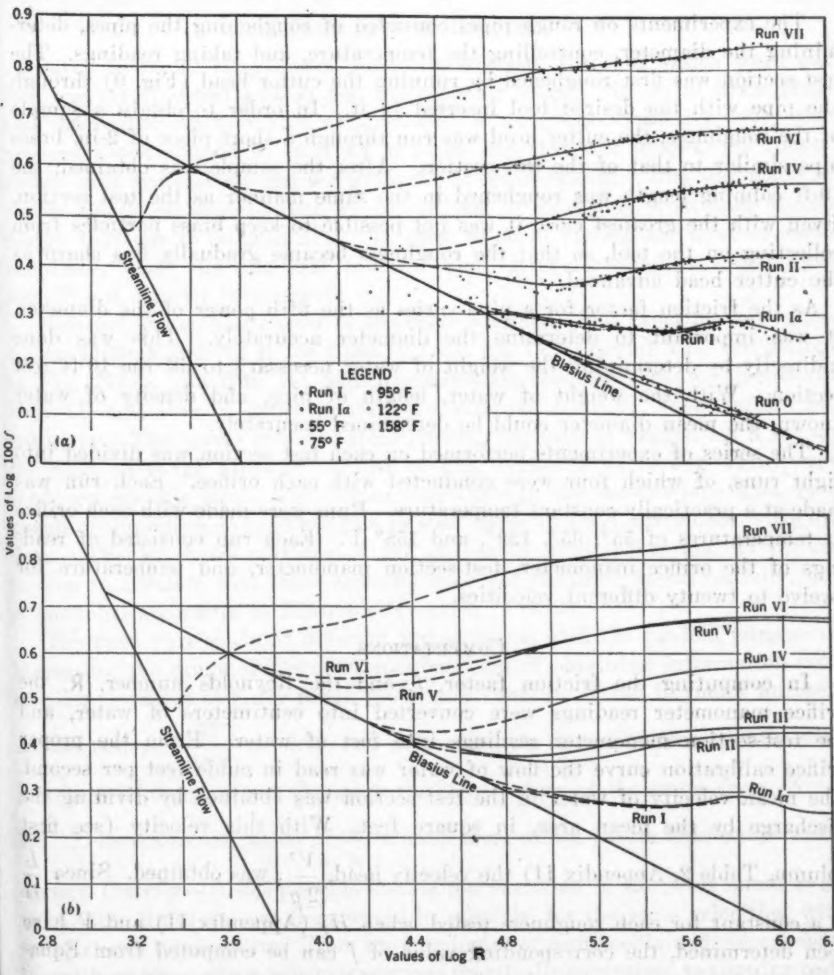

FIG. 11.—EXPERIMENTAL DATA: (a) FOR RUNS 0, I, Ia, II, IV, VI, AND VII; (b) CURVES OF f PLOTTED AGAINST R OBTAINED FROM TESTS ON ROUGHENED PIPES.

manometer to be small compared with the pressure drop at the same velocity that would occur between the two ends of the test section. Accordingly, it was not necessary to add a correction to the pressure-drop measurements.

Tests were made on smooth pipes in order to obtain a basis for comparison with other investigators, and also to enable operators to become familiar with the equipment. Two sets of experiments were made with drawn brass pipe to obtain curves showing the relation of f to the Reynolds number for different temperatures (see Run 0, Fig. 11(a)). These runs were conducted in the same manner as those on rough pipe.

Experiments for Obtaining Data

The experiments on rough pipes consisted of roughening the pipes, determining the diameter, controlling the temperature, and taking readings. The test section was first roughened by running the cutter head (Fig. 9) through the pipe with the desired tool inserted in it. In order to obtain a sample of the roughness, the cutter head was run through a short piece of 2-in. brass pipe similar to that of the test section. After the sample was obtained, the 10-ft calming length was roughened in the same manner as the test section. Even with the greatest care, it was not possible to keep brass particles from collecting on the tool, so that the roughness became gradually less sharp as the cutter head advanced.

As the friction factor for a pipe varies as the fifth power of the diameter, it was important to determine the diameter accurately. This was done indirectly by determining the weight of water necessary to fill the 10-ft test section. With the weight of water, length of pipe, and density of water known, the mean diameter could be determined accurately.

The series of experiments performed on each test section was divided into eight runs, of which four were conducted with each orifice. Each run was made at a practically constant temperature. Runs were made with each orifice at temperatures of 55°, 95°, 122°, and 158° F. Each run consisted of readings of the orifice manometer, test-section manometer, and temperature for twelve to twenty different velocities.

Computations

In computing the friction factor, f, and the Reynolds number, R, the orifice manometer readings were converted into centimeters of water, and the test-section manometer readings into feet of water. From the proper orifice calibration curve the flow of water was read in cubic feet per second. The mean velocity of water in the test section was obtained by dividing the discharge by the mean area, in square feet. With this velocity (see first column, Table 2, Appendix II) the velocity head, $\frac{V^2}{2g}$, was obtained. Since $\frac{L}{D}$ is a constant for each roughness tested, when H_f (Appendix II) and V have been determined, the corresponding value of f can be computed from Equation (4) for each set of observations. The fraction, $\frac{\rho}{\mu}$, contained in the

Reynolds number is a function of the temperature, and, therefore, values of T can be traced through corresponding values in the third column of Table 2 (Appendix II). A table of values of $\frac{\rho}{\mu}$, in seconds per feet^2, was made up[14] for each 0.1° C from 10 to 75 degrees. With V in feet per second, D, in feet, and $\frac{\rho}{\mu}$, in seconds per feet^2, the Reynolds number is seen to be dimensionless, and in consistent units. Finally, values of R and f, can be computed from the values listed in the fourth and fifth columns of Table 2 (see Appendix II).

Discussion of Results

Seven artificial roughnesses were investigated, three of which were cut into one set (the test section and the calming length, Fig. 3) of brass pipes and the remaining four into another set. A roughness typified by very fine grooves was first used in each set of pipes and this was followed by roughnesses with successively coarser grooves. In each case an effort was made to choose roughness designs that would conform as closely as practicable to the design that was to be used next. In spite of every precaution, however, each roughness made the one that followed more irregular than it would have been if cut directly into smooth pipe.

Relation of Friction Factor to Reynolds Number.—The experimental values of log (100 f) and log R for Runs 0, I, I*a*, II, IV, VI, and VII, are plotted in Fig. 11(*a*). As Run III had approximately the same friction factors as Run II, and Run V approximately the same as Run VI, their results are not given. The curves obtained from plotting log (100 f) against log R for all the runs on rough pipes are shown in Fig. 11(*b*). Run 0, Fig. 11(*a*), was made on smooth pipe. The dotted line through the smooth-pipe data is a graph of Equation (7). This line is seen to be in good agreement with the data. The left-hand portions of the curves of Figs. 11(*a*) and 11(*b*) are broken to indicate that these sections were not obtained experimentally, but constructed according to Nikuradse's curves (Fig. 2).

The runs were made at different temperatures with the twofold purpose of increasing the range of Reynolds numbers and of obtaining information concerning the value of f, for rough pipes, as a function of the Reynolds number. As the ratio, $\frac{\mu\,(55°)}{\mu\,(158°)} = 2.98$, and as the density decreases very slightly with increases in temperature, changing the temperature of the water from 55° to 158° F increases the value of $\frac{\rho}{\mu}$ almost three times. With this temperature change, therefore, it was possible to obtain a variation in Reynolds numbers almost three times greater than would be possible, for the same velocities, with water at one temperature. From the standpoint of results obtained, experimenting with water at these temperatures was practically equivalent to running tests

[14] The values of ρ and μ were taken from "Elements of Chemical Engineering," by Badger and McCabe.

with other liquids which had corresponding values of $\frac{\rho}{\mu}$. To obtain the values indicated by any of the points on the curves shown in Fig. 11(*a*) would require velocities about three times as great if tests were run at a temperature of 55° F as would be required at a temperature of 158° F. In spite of the change in impact brought about by this velocity change, the friction factor remains the same. As comparatively few experiments have been conducted on artificially roughened pipes with viscosities other than those of water at temperatures between 32° and 75° F, these tests over a greater viscosity range furnish additional proof that, for a given rough pipe, f is a function of the Reynolds number.

Photo-Micrographic Study of the Roughnesses.—As a means of determining as closely as possible the shape of grooves in each test section, photo-micrographs, magnified to four times the actual size, of the pipe sample and cutting tool were taken for each roughness. In Figs. 13 and 14, photo-micrographs are magnified to 2.94 times their actual size. In some cases a groove intermediate between that of the cutting tool and the sample seemed to represent best the form of the roughness. In other cases (depending upon the number of previous cuts in the pipe, the wear on the cutting tool, and the clearness of the photo-micrographs), the roughness appeared to be better represented either by the sample or by the tool. The number used herein to designate a type of roughness, corresponds to that used in Figs. 11(*a*) and

TABLE 1.—COMPARISON OF VALUES OF k' WITH ROUGHNESS DIMENSIONS

Roughness (see Fig. 12) (1)	f (= a constant) (2)	$\log \frac{r}{k'}$ (3)	Radius, r, in inches (4)	Radius of sand grains, k', in inches (5)	Average depth of grooves, in inches (6)	Number of grooves per inch (7)	Roughness (see Fig. 12) (1)	f (= a constant) (2)	$\log \frac{r}{k'}$ (3)	Radius, r, in inches (4)	Radius of sand grains, k', in inches (5)	Average depth of grooves, in inches (6)	Number of grooves per inch (7)
I........	0.0188	2.775	1.036	0.0017	0.005	46	IV.......	0.0370	1.727	1.043	0.0196	0.012	23
I*a*........	0.0188	2.775	1.036	0.0017	0.005	46	V.......	0.0465	1.45	1.043	0.0370	0.012	23
II........	0.0266	2.195	1.034	0.0066	0.005	23	VI.......	0.0473	1.43	1.047	0.0390	0.016	11.5
III.......	0.0272	2.155	1.035	0.0072	0.005	23	VII.....	0.0700	1.02	1.052	0.1005	0.022	11.5

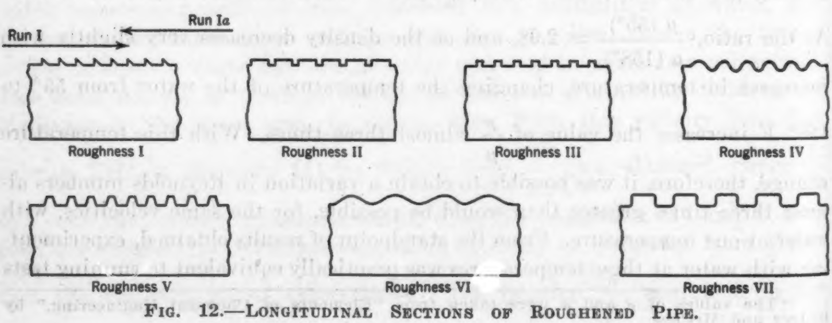

FIG. 12.—LONGITUDINAL SECTIONS OF ROUGHENED PIPE.

11(b) to indicate the run in which this roughness was used. Essential dimensions are classified in Table 1 and the roughness is shown diagrammatically for each case, in Fig. 12.

Roughness I. — The photo-micrographs of the sample and the tool for Roughness I are shown in Figs. 13(a) and 13(b), respectively. This was the

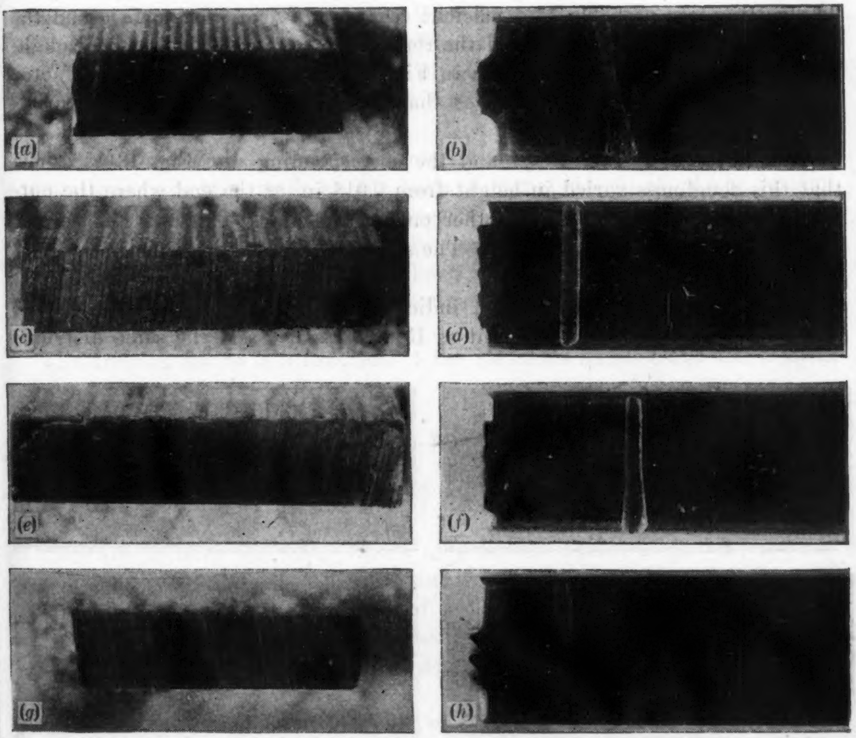

FIG. 13.—PHOTO-MICROGRAPHS OF LONGITUDINAL SECTIONS OF PIPE AND CUTTING TOOLS: a, c, e, AND g, RESPECTIVELY, DENOTE ROUGHNESSES I, II, III AND IV, WITH b, d, f, AND g, THE CORRESPONDING CUTTING DIES.

first cut through the first set of pipes, and, also, the finest roughness. Unfortunately, the sample photo-micrograph (Fig. 13(a)) does not show the profile of the roughness in such a way that measurements can be taken from it. The cutting tool (Fig. 13(b)), also, was badly damaged in going through the calming length, which had been slightly deformed as a result of some mishandling in transit. The arrows in Fig. 12 show the direction of flow. Run Ia was made with the same roughness, but with the direction of flow reversed.

Roughness II.—The photo-micrographs for Roughness II are shown in Fig. 13(c) and the cutting die in Fig. 13(d). It was the first cut into the second set of brass pipes, and, therefore, the shape and dimensions of

the roughness can be taken from the tool. The photo-micrograph of the sample does not aid much in obtaining the dimensions of the elements.

Roughness III.—The roughness indicated by Fig. 13(*e*) and the corresponding die, Fig. 13(*f*), was obtained by cutting over Roughness II with a tool designed to leave the roughness shown in Fig. 12. This was the only case of a roughness being cut to the same depth as that which preceded it. The cutting was probably 75% efficient (that is, 75% of pipe surface had the roughness shown by the tool, and the remainder was characterized by Roughness II). Roughness III as shown in Fig. 11(*b*) had a higher friction factor than Roughness II. This indicates that a projection causes greater turbulence than a groove.

Roughness IV.—Fig. 13(*g*) and the corresponding die, Fig. 13(*h*), show that this roughness varied in height from 0.018 in. at the end where the cutting began, to 0.006 in. at the other end of the pipe. This was the second roughness in the first set of pipes. The shape in Fig. 12 was a mean between tool and sample.

Roughness V.—The roughness, indicated by Fig. 14(*a*) and Fig. 14(*b*), had about the same average height as Roughness IV, and the same distribu-

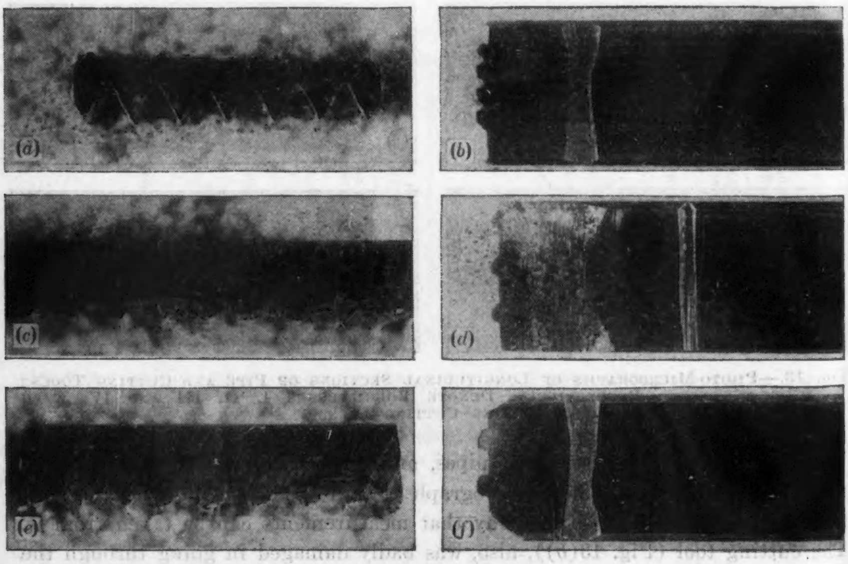

FIG. 14.—PHOTO-MICROGRAPHS OF LONGITUDINAL SECTIONS OF PIPE AND CUTTING TOOLS: *a*, *c*, AND *e*, RESPECTIVELY, DENOTE ROUGHNESSES V, VI AND VII, WITH *b*, *d*, AND *e*. THE CORRESPONDING CUTTING DIES.

tion, but was much more rough and abrupt in shape. This was the third roughness in the second set of pipes. The effect of shape is strikingly shown by the difference in friction factors of Roughnesses IV and V. In Fig. 14(*a*), the threads on the lower side are the standard for 2-in. pipes.

Roughness VI.—The third and last roughness of the first set of pipes (Roughness VI) is shown in Fig. 14(*c*) and Fig. 14(*d*). The shape was wavy in character, without sharp projections, as shown in Fig. 12. This roughness had a slightly greater friction factor than Roughness V, which had smaller elements and with twice as many peaks per inch. The decidedly abrupt shape of Roughness V seems to account for its friction factor being almost as large as that of Roughness VI.

Roughness VII.—Fig. 14(*e*) and Fig. 14(*f*) show Roughness VII, the fourth cut through the second set of pipes. As would be expected with its large grooves and sharp projections, this roughness gave the greatest friction factor of any shape investigated.

Relative Roughness.—While the description of the roughnesses and their comparisons have been made on the basis of actual size, it should be remembered that this can be done only because the radius of the pipe has changed little in comparison with the changes in roughness. This investigation neither proves nor disproves the similitude theory as applied to relative roughness. Nikuradse has made a satisfactory experimental demonstration of the validity of this theory.

The friction factor is a function of the Reynolds number and the relative roughness, or,

$$f = \phi\left(\mathsf{R}, \frac{k}{r} \right) \qquad (11)$$

The relative roughness factor, $\frac{k}{r}$, must have the same value for any two pipes if they are to have the same values of f for all values of R. At present, there is no known method of determining k. It is undoubtedly some function of the size, shape, and distribution of individual roughness elements. The problem of determining numerical values of k that will define certain standard surfaces is an important one that will doubtless receive the attention of future investigators.

Comparison of Results with Those of Other Investigators.—A comparison with Nikuradse's experiments was made by investigating conditions in the region of large Reynolds numbers where the curve representing the mean of his results is approximately horizontal. Values of k' were computed from Nikuradse's formula (Equation (10)), the friction factor, f, being taken from the right side of the curves in Fig. 11(*b*). These values of k' are the diameters of sand grains required to give the same roughness effect as the corresponding grooves. Table 1 gives the value of k' obtained in this manner. In all cases, except Roughness I and Roughness I*a*, the diameters of sand grains are larger than the depths of the grooves. The shape of the rough surfaces seems to have almost as much effect as the depth of the grooves. In Runs I and I*a* the closeness of the roughness particles appears to account for the small friction factor. Run III has the same depth of groove as Run I, and the shape is not radically different; and yet the sand grains for Run III have about four times the diameter of those for Run I. Apparently, this increase in the value of f is caused entirely by the distribution of the roughness elements.

The ratio of the depth of groove to the radius of the pipe in Schiller's experiments was larger in all cases than those used in this investigation. The distributions used by Schiller were about 32 and 64 grooves per in. Because of the relatively larger grooves and the smaller range of Reynolds numbers covered by his experiments, it is difficult to establish relationships between the results of the two investigations.

Acknowledgments

The writer wishes to express his grateful appreciation to H. W. King, M. Am. Soc. C. E., and to Professor W. L. Badger, of the University of Michigan, for their interest and suggestions during this investigation; and to Mr. Weyburn M. Dodge for his aid in constructing the equipment.

Conclusion

The experiments described herein indicate that the shape of the roughness elements is quite as important as the size. This is evident from an examination of Columns (5) and (6) of Table 1, as well as Fig. 11. Runs I, II, and III, while having the same depth of groove, have a wide variation in friction factor. As the shape of Runs I and III are somewhat similar, the difference in friction factor may be attributed to the difference in distribution of the roughness elements.

APPENDIX I

Notation

The following symbols, adopted for use in this paper, conform in all essential respects with "Symbols for Hydraulics"[2] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1929:

- c = coefficient; a constant for each type of roughened surface (not to be confused with Chezy's coefficient, C).
- d = depth; depth of flow; depth of section.
- f = friction factor used in expressing loss of head in pipes; Darcy's friction factor; as a subscript, f denotes "friction."
- g = gravity constant; acceleration due to gravity.
- h = head; as a subscript, h denotes "hydraulic radius."
- k = a measure of absolute roughness; k' = diameter of sand grains, taken as a measure of absolute roughness.
- r = radius of pipe
- A = area; cross-section area of pipe
- C = Chezy's coefficient (not to be confused with c, as introduced in the paper).
- D = mean interior diameter of a pipe.
- F = force; total pressure; F_l = loss of pressure.
- H = total head; H_f = total loss of head due to friction.

L = length of experimental pipe sections.
R = hydraulic radius (not to be confused with R (see Equation (5)).
R = Reynolds number; R_h = Reynolds number based on the hydraulic radius
T = temperature.
V = velocity; average velocity in a section.
μ = viscosity, absolute.
γ = viscosity, kinetic $= \dfrac{\mu}{\rho}$.
ρ = density
ϕ = function of.

APPENDIX II

Summary of Computations

TABLE 2.—Computations to Determine Friction Factor, f, and Reynolds Number, R

Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	log R	log (100 f)	Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	log R	log (100 f)
(a) Run 0, Part a; A = 0.0233 Square Feet; D = 0.1722 Feet; and $\frac{L}{D}$ = 55.152.									
5.71	0.555	74 800	4.867	0.2975	18.19	3.99	126 440	5.598	0.148
1.85	0.059	74 800	4.377	0.3035	19.44	4.52	126 960	5.629	0.144
1.39	0.0514	74 800	4.253	0.492	20.35	4.94	127 740	5.651	0.143
3.14	0.159	74 000	4.602	0.274	21.2	5.34	127 740	5.669	0.145
3.84	0.26	73 200	4.685	0.314	22.15	5.74	128 260	5.69	0.135
4.243	0.333	78 800	4.760	0.334	22.8	6.06	128 580	5.704	0.136
5.45	0.488	80 060	4.876	0.282	24.36	6.85	129 300	5.734	0.132
5.895	0.570	80 480	4.9120	0.282	26.4	7.95	129 820	5.772	0.124
9.69	1.275	125 400	5.322	0.200	28.2	8.89	130 340	5.802	0.114
11.71	1.775	125 400	5.44	0.178	29.4	9.64	130 600	5.820	0.1125
13.13	2.175	125 400	5.454	0.166	30.45	10.2	130 600	5.836	0.108
14.59	2.635	125 400	5.499	0.159	32.38	11.4	130 860	5.862	0.104
15.75	3.05	125 660	5.532	0.156	34.1	12.57	130 860	5.886	0.101
16.3	3.21	125 660	5.548	0.1495	35.6	13.5	131 120	5.906	0.095
17.2	3.61	125 920	5.572	0.153	38.6	15.37	131 380	5.941	0.08

TABLE 2.—(*Continued*)

Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	Values of: log R	Values of: log (100 f)	Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	Values of: log R	Values of: log (100 f)
(*b*) Run 0, Part *b*; $A = 0.0234$ Square Feet; $D = 0.1720$ Feet; and $\frac{L}{D} = 55.23$									
2.38	0.0945	99 550	4.610	0.288	7.19	0.5535	178 340	5.344	0.096
3.717	0.2224	100 700	4.808	0.273	8.72	0.878	178 340	5.428	0.128
4.96	0.3855	100 700	4.9345	0.2595	12.85	1.831	177 800	5.594	0.112
5.395	0.454	100 700	4.970	0.2595	17.08	3.186	177 800	5.718	0.1045
5.99	0.547	100 470	5.0144	0.249	19.86	4.273	178 340	5.784	0.102
6.64	0.6575	100 470	5.0593	0.240	23.56	5.909	177 800	5.858	0.094
7.67	0.850	100 700	5.1233	0.226	27.4	7.75	177 800	5.924	0.081
8.71	1.090	100 940	5.180	0.224	30.8	9.86	177 800	5.974	0.084
9.68	1.328	100 940	5.225	0.218	34.7	11.95	177 800	6.026	0.062
10.87	1.628	100 940	5.2755	0.2055	36.6	13.09	177 800	6.050	0.056
9.51	1.27	99 550	5.212	0.2135	39.6	14.49	177 800	6.090	0.031
13.20	2.31	99 320	5.353	0.205	8.68	0.911	229 000	5.534	0.150
15.93	3.24	99 550	5.436	0.172	13.5	2.02	229 810	5.728	0.111
18.42	4.16	99 780	5.50	0.155	17.34	3.255	229 540	5.836	0.100
21.52	5.69	99 780	5.3875	0.148	19.65	4.07	229 270	5.890	0.088
23.95	6.65	99 320	5.612	0.130	22.23	5.122	229 000	5.922	0.0795
27.36	8.45	99 090	5.668	0.120	24.0	5.85	229 270	5.976	0.073
30.20	10.30	99 550	5.7138	0.1195	27.44	7.295	229 540	6.0355	0.052
32.7	11.97	99 780	5.749	0.115	30.43	8.81	229 540	6.080	0.440
34.86	13.43	99 320	5.7754	0.1096	33.43	10.60	229 270	6.124	0.042
36.7	14.57	99 320	5.797	0.160	36.27	12.18	229 270	6.159	0.034
37.88	15.20	99 550	5.812	0.0918	38.62	13.60	229 810	6.194	0.027
8.90	0.975	170 240	5.416	0.156	6.84	0.625	232 700	5.437	0.192
12.78	1.918	169 700	5.572	0.136	8.28	0.907	233 300	5.522	0.198
16.36	3.028	169 700	5.679	0.120	13.71	2.163	232 000	5.738	0.127
18.8	3.97	169 700	5.740	0.1175	16.98	3.181	231 180	5.830	0.1105
21.2	5.00	169 700	5.792	0.112	20.47	4.506	231 730	5.912	0.098
23.33	5.885	169 970	5.834	0.100	23.6	5.790	232 700	5.976	0.082
26.58	7.40	170 240	5.892	0.088	27.6	7.41	232 700	6.044	0.055
29.8	9.045	169 700	5.940	0.074	30.92	9.10	232 700	6.093	0.0455
32.5	10.8	169 430	5.977	0.076	34.5	10.97	232 700	6.144	0.030
34.8	12.11	169 700	6.008	0.067	37.1	12.30	232 700	6.176	0.017
37.2	13.44	169 700	6.042	0.055	39.6	13.71	232 000	6.203	0.026
39.55	14.50	169 700	6.062	0.034	.....	......	....	......	.....
(*c*) Run I: $A = 0.2333$ Square Feet; $D = 0.1727$ Feet; and $\frac{L}{D} = 54.99$.									
1.20	0.0265	75 600	4.195	0.332	9.88	1.567	91 600	5.193	0.2735
1.72	0.0625	76 000	4.354	0.393	10.43	1.747	91 600	5.218	0.272
2.35	0.1025	74 800	4.482	0.337	10.9	1.89	91 600	5.236	0.270
2.785	0.138	80 900	4.590	0.318	9.68	1.554	100 010	5.2214	0.2880
3.01	0.1595	83 210	4.636	0.314	10.80	1.90	99 780	5.2669	0.2788
3.43	0.203	85 100	4.703	0.3055	11.59	2.185	100 470	5.3032	0.2788
3.93	0.263	85 730	4.765	0.300	12.94	2.70	100 700	5.3522	0.2742
4.31	0.321	85 940	4.806	0.305	14.11	3.195	100 700	5.3838	0.2748
4.61	0.366	86 300	4.838	0.303	15.48	3.815	101 180	5.4322	0.2718
4.91	0.409	86.780	4.867	0.296	17.11	4.63	102 140	5.4800	0.2648
5.09	0.442	87 200	4.884	0.300	18.58	5.43	102 860	5.517	0.2648
5.51	0.5165	87 640	4.922	0.299	19.26	5.84	103 100	5.5353	0.2648
5.99	0.605	88 300	4.960	0.296	20.36	6.55	104 300	5.5697	0.2672
6.63	0.7305	89 180	5.009	0.290	21.56	7.44	105 020	5.5916	0.2768
7.20	0.858	89 620	5.046	0.288	23.07	8.45	105 740	5.6243	0.2695
7.54	0.934	90 060	5.068	0.284	25.18	10.2	106 220	5.6646	0.2742
7.96	1.02	90 500	5.095	0.2755	27.05	11.82	106 940	5.6981	0.2765
8.43	1.15	90 720	5.120	0.278	29.12	13.6	107 660	5.7332	0.2742
8.80	1.254	91 160	5.142	0.278	30.73	15.18	107 900	5.7574	0.2742
9.27	1.39	91 380	5.164	0.2765	.....	......	....	......	.....
(*d*) Run I*a*: $A = 0.2333$ Square Feet; $D = 0.1727$ Feet; and $\frac{L}{D} = 54.99$									
9.65	1.556	110 400	5.265	0.290	19.4	6.08	115 150	5.587	0.2765
11.28	2.073	110 900	5.334	0.2815	20.63	6.77	115 400	5.615	0.2695
13.21	2.818	111 900	5.407	0.276	21.76	7.55	115 650	5.638	0.272
14.78	3.46	112 150	5.457	0.268	22.6	8.14	116 900	5.658	0.272
16.15	4.16	112 900	5.498	0.2705	23.4	8.80	117 400	5.676	0.2735
17.31	4.79	113 400	5.531	0.2705	24.2	9.41	117 900	5.692	0.274
18.35	5.39	114 150	5.558	0.272	25.1	10.45	118 900	5.712	0.288

TABLE 2.—(*Continued*)

Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	Values of: log **R**	Values of: log (100 f)	Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	Values of: log **R**	Values of: log (100 f)
(*d*) Run I*a*: $A = 0.2333$ Square Feet; $D = 0.1727$ Feet; and $\frac{L}{D} = 54.99$. (*Continued*)									
26.4	11.45	119 650	5.737	0.2835	9.1	1.336	119 150	5.272	0.276
27.78	12.69	120 150	5.760	0.284	8.28	1.130	121 400	5.240	0.2855
28.75	13.62	120 400	5.776	0.286	1.653	0.0515	74 200	4.326	0.343
29.65	14.48	121 650	5.794	0.298	7.60	0.922	144 380	5.278	0.272
30.55	15.45	122 400	5.810	0.288	7.555	0.908	152 700	5.299	0.270
31.5	16.40	123 150	5.826	0.286	10.72	1.801	160 500	5.473	0.263
32.4	17.15	124 900	5.844	0.281	10.67	1.785	180 500	5.522	0.265
32.93	17.91	130 340	5.870	0.287	10.57	1.760	196 080	5.552	0.266
32.93	17.88	132 940	5.877	0.285	10.53	1.750	201 680	5.564	0.266
14.43	3.25	130 600	5.513	0.261	9.34	1.437	85 310	5.138	0.285
17.66	4.89	142 820	5.638	0.263	11.07	2.023	86 780	5.219	0.287
21.28	7.245	147 500	5.734	0.272	13.17	2.835	87 200	5.297	0.282
24.3	9.54	154 000	5.810	0.276	15.7	3.964	94 260	5.408	0.276
26.27	11.29	157 900	5.855	0.281	17.82	5.144	100 010	5.489	0.277
28.21	13.20	161 800	5.896	0.287	18.97	5.815	104 300	5.534	0.278
33.04	16.92	167 540	5.982	0.258	20.3	6.645	128 000	5.652	0.275
30.55	14.26	174 020	5.962	0.252	22.0	7.812	137 880	5.718	0.276
33.12	17.17	185 160	6.026	0.2625	23.5	9.25	143 860	5.766	0.293
2.763	0.141	112 650	4.731	0.3335	33.64	17.84	151 660	5.946	0.265
3.87	0.2605	113 400	4.88	0.308	27.15	11.82	157 380	5.868	0.274
4.705	0.3741	112 650	4.961	0.296	29.8	14.16	162 320	5.922	0.2695
5.15	0.448	114 900	5.009	0.296	31.62	16.04	172 670	5.976	0.2735
5.58	0.506	114 650	5.042	0.280	33.6	17.91	182 660	6.027	0.2695
6.025	0.584	114 650	5.076	0.2755	33.77	17.40	194 960	6.058	0.252
7.25	0.866	115 150	5.098	0.286	25.62	10.30	199 440	5.946	0.262
7.506	0.931	115 400	5.175	0.286	33.78	17.29	205 040	6.08	0.2495
11.0	1.911	115 650	5.342	0.266	33.78	17.30	215 800	6.102	0.250
10.5	1.778	115 650	5.322	0.275	33.77	17.41	221 800	6.114	0.2525
9.96	1.611	117 150	5.305	0.278	33.78	17.50	224 700	6.120	0.255
(*e*) Run II: $A = 0.02331$ Square Feet; $D = 0.1723$ Feet; and $\frac{L}{D} = 55.14$.									
10.69	2.25	74 800	5.140	0.3615	12.49	3.32	128 000	5.440	0.396
9.80	1.90	75 000	5.103	0.3625	10.2	2.11	128 520	5.355	0.384
9.0	1.61	75 400	5.068	0.3655	8.34	1.442	128 000	5.265	0.3825
8.145	1.319	75 600	5.026	0.3645	10.42	2.286	128 000	5.362	0.3895
6.93	0.97	76 000	4.952	0.372	9.77	2.002	129 040	5.337	0.389
5.825	0.6845	76 600	4.886	0.372	8.98	1.662	129 040	5.300	0.3815
5.107	0.545	76 800	4.830	0.388	7.85	1.242	128 780	5.242	0.372
4.197	0.372	77 000	4.745	0.392	6.96	0.955	128 260	5.188	0.362
3.22	0.2192	77 400	4.634	0.392	5.97	0.684	127 480	5.119	0.3505
2.143	0.107	78 000	4.460	0.435	5.08	0.496	127 740	5.048	0.3505
1.582	0.0613	78 400	4.330	0.456	4.073	0.3185	128 000	4.954	0.3505
2.575	0.1421	79 010	4.545	0.398	2.96	0.174	128 000	4.815	0.364
3.615	0.2665	79 220	4.692	0.3765	2.283	0.1065	128 000	4.703	0.378
4.70	0.4504	78 800	4.806	0.376	3.482	0.232	128 000	4.886	9.348
5.505	0.615	78 400	4.872	0.374	4.565	0.3945	128 000	5.004	0.344
33.4	24.95	76 000	5.642	0.416	5.55	0.5905	128 000	5.088	0.350
31.6	22.1	76 200	5.618	0.411	10.6	2.44	167 540	5.486	0.4035
29.76	19.5	76 400	5.593	0.410	9.85	2.092	166 480	5.452	0.400
28.0	17.2	76 600	5.568	0.4075	9.04	1.73	166 220	5.414	0.392
25.95	14.61	76 800	5.536	0.404	8.025	1.34	166 480	5.363	0.386
24.18	12.54	76 800	5.505	0.3985	7.04	1.011	166 740	5.306	0.3765
22.23	10.63	77 000	5.470	0.400	6.09	0.7445	167 000	5.244	0.369
20.36	8.82	77 400	5.434	0.396	3.91	0.300	166 220	5.051	0.360
18.4	7.125	77 600	5.391	0.391	2.86	0.1665	165 180	4.911	0.375
16.52	5.72	77 800	5.346	0.388	2.122	0.100	164 920	4.781	0.414
14.47	4.33	77 800	5.288	0.383	3.462	0.228	164 920	4.994	0.346
12.58	3.276	78 000	5.228	0.3825	4.445	0.381	165 700	5.104	0.352
10.86	2.403	78 000	5.165	0.376	5.505	0.599	165 960	5.197	0.362
9.35	1.753	78 200	5.10	0.370	33.9	26.06	167 000	5.989	0.422
33.43	25.62	128 520	5.870	0.426	32.1	23.4	167 000	5.966	0.424
32.0	23.42	128 520	5.851	0.425	29.86	20.2	167 000	5.934	0.4215
29.7	19.7	128 000	5.816	0.416	27.36	17.02	167 000	5.896	0.424
27.2	16.54	128 520	5.781	0.416	25.2	14.37	167 000	5.860	0.422
24.15	12.94	128 000	5.726	0.414	22.75	11.79	167 000	5.816	0.4255
21.6	10.4	128 520	5.6805	0.415	19.9	9.07	167 000	5.758	0.428
19.36	8.30	128 520	5.632	0.413	18.11	7.43	167 000	5.717	0.422
17.1	6.39	128 000	5.577	0.408	15.75	5.535	167 000	5.656	0.4145
14.7	4.64	128 000	5.511	0.398	13.21	3.82	167 000	5.580	0.4065

TABLE 2.—(*Continued*)

Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	Values of: log R	Values of: log (100 f)	Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	Values of: log R	Values of: log (100 f)
(*e*) Run II: $A = 0.02331$ Square Feet; $D = 0.1723$ Feet; and $\frac{L}{D} = 55.14$. (*Continued*)									
11.53	2.886	167 000	5.521	0.402	10.6	2.53	221 700	5.608	0.4185
9.64	2.027	167 000	5.443	0.405	9.84	2.157	222 900	5.578	0.414
34.0	26.02	222 900	6.116	0.420	9.00	1.778	222 900	5.540	0.408
32.2	23.24	222 900	6.093	0.418	7.975	1.372	222 900	5.487	0.402
30.4	20.62	222 900	6.068	0.4165	6.94	1.010	222 900	5.426	0.3895
27.7	17.35	222 900	6.028	0.420	6.02	0.757	222 900	5.365	0.388
25.6	14.87	222 900	5.994	0.422	5.11	0.532	222 900	5.294	0.376
23.55	12.68	222 900	5.957	0.427	4.005	0.318	222 900	5.188	0.364
21.44	10.6	222 900	5.917	0.429	2.82	0.160	222 900	5.036	0.3705
19.44	8.75	222 900	5.874	0.432	1.902	0.0825	222 900	4.865	0.4255
17.6	7.16	222 900	5.831	0.431	3.55	0.244	223 200	5.136	0.354
15.3	5.345	222 900	5.776	0.424	4.51	0.4045	223 200	5.240	0.364
13.17	3.94	222 900	5.704	0.424	5.595	0.6455	222 900	5.332	0.381
11.11	2.777	222 900	5.631	0.418	.....	....	....	....	....
(*f*) Run III: $A = 0.02336$ Square Feet; $D = 0.1725$ Feet; and $\frac{L}{D} = 55.09$.									
31.96	23.77	74 600	5.614	0.434	5.58	0.631	128 000	5.091	0.374
30.2	21.19	74 800	5.590	0.434	6.02	0.736	128 000	5.124	0.376
28.55	18.7	75 200	5.580	0.428	32.02	24.11	166 220	5.964	0.4365
26.58	16.1	75 600	5.538	0.426	30.35	21.38	166 480	5.940	0.433
24.46	13.58	76 000	5.507	0.422	28.4	18.69	166 480	5.912	0.432
22.45	11.45	76 200	5.470	0.424	26.46	16.16	167 000	5.882	0.430
20.55	9.505	76 400	5.433	0.420	24.02	13.40	166 480	5.839	0.433
18.53	7.69	76 600	5.388	0.418	22.0	11.37	166 480	5.801	0.437
16.4	5.91	77 000	5.339	0.410	20.0	9.40	166 740	5.760	0.438
14.21	4.48	77 400	5.277	0.413	18.09	7.61	166 740	5.716	0.435
11.36	2.819	77 600	5.183	0.407	16.0	5.89	166 740	5.663	0.430
9.07	1.74	77 800	5.086	0.393	13.5	4.192	167 000	5.590	0.428
10.77	2.442	77 800	5.160	0.392	11.36	2.957	167 000	5.515	0.428
10.06	2.13	77 800	5.130	0.390	9.8	2.186	166 740	5.450	0.4245
9.245	1.771	77 800	5.094	0.384	9.2	1.928	166 480	5.423	0.424
8.43	1.473	77 800	5.054	0.384	10.74	2.655	169 700	5.498	0.428
7.55	1.17	78 000	5.006	0.3795	10.07	2.281	169 160	5.468	0.420
6.55	0.875	78 200	4.946	0.378	9.19	1.874	168 350	5.426	0.414
5.495	0.62	78 600	4.870	0.381	8.243	1.48	168 350	5.378	0.406
4.56	0.4345	78 600	4.790	0.387	7.225	1.149	168 350	5.321	0.410
3.74	0.2922	78 800	4.706	0.388	6.195	0.796	167 270	5.252	0.3845
2.87	0.174	79 010	4.592	0.392	5.20	0.571	167 270	5.176	0.392
2.078	0.087	79 220	4.454	0.372	4.126	0.359	167 270	5.076	0.3905
1.35	0.0467	79 430	4.267	0.476	3.083	0.1988	167 270	4.95	0.3875
1.628	0.0639	80 060	4.352	0.450	1.964	0.0890	167 540	4.754	0.431
2.483	0.1251	80 900	4.540	0.375	1.61	0.0624	167 540	4.668	0.449
3.295	0.216	80 900	4.662	0.366	4.72	0.4468	167 000	5.133	0.369
4.16	0.356	80 480	4.672	0.380	5.725	0.672	165 700	5.214	0.380
5.10	0.5395	80 060	4.848	0.3845	32.4	24.22	222 900	6.096	0.431
32.0	24.2	126 180	5.844	0.4415	30.73	21.61	221 400	6.070	0.428
30.08	21.33	127 480	5.821	0.439	28.82	19.29	221 400	6.042	0.432
28.22	18.51	128 000	5.795	0.4325	27.12	17.03	222 900	6.018	0.432
25.83	15.41	128 260	5.758	0.430	24.83	14.37	222 900	5.980	0.434
22.72	12.18	128 260	5.701	0.440	23.0	12.38	222 900	5.947	0.436
20.81	10.31	128 000	5.663	0.444	20.86	10.29	222 900	5.905	0.440
18.75	8.345	128 000	5.617	0.443	18.58	8.14	222 900	5.854	0.440
16.81	6.64	128 000	5.570	0.438	16.54	6.48	222 900	5.804	0.442
14.73	5.055	128 000	5.513	0.434	14.0	4.577	222 900	5.732	0.435
12.28	3.48	128 000	5.433	0.4315	11.7	3.218	222 900	5.654	0.438
10.43	2.492	128 000	5.353	0.426	10.89	2.727	224 280	5.626	0.4295
9.325	0.978	128 000	5.314	0.4235	10.07	2.323	223 420	5.589	0.428
10.67	2.522	127 480	5.370	0.414	9.30	1.978	221 400	5.550	0.426
9.71	2.10	129 820	5.338	0.415	8.29	1.549	221 400	5.501	0.420
8.94	1.733	129 820	5.301	0.4035	7.20	1.141	221 400	5.439	0.412
8.095	1.416	129 300	5.257	0.402	6.17	0.846	221 400	5.373	0.4145
7.05	1.052	129 300	5.197	0.393	5.20	0.585	221 400	5.298	0.403
6.02	0.7355	128 520	5.126	0.376	4.163	0.3625	222 000	5.203	0.388
4.992	0.505	128 000	5.042	0.374	3.303	0.2325	222 900	5.104	0.3965
3.92	0.302	128 000	4.937	0.361	2.571	0.1372	222 900	4.995	0.385
2.857	0.1549	128 000	4.800	0.346	1.893	0.081	222 900	4.862	0.422
2.32	0.108	128 520	4.712	0.370	3.67	0.2656	222 900	5.150	0.362
3.437	0.224	128 520	4.882	0.346	4.69	0.469	223 420	5.257	0.3965
4.425	0.396	128 780	4.992	0.373	5.745	0.720	222 900	5.344	0.4065

TABLE 2.—(*Continued*)

Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	Values of: log R	Values of: log (100 f)	Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	Values of: log R	Values of: log (100 f)
(*g*) Run IV: $A = 0.02375$ Square Feet; $D = 0.1739$ Feet; and $\frac{L}{D} = 54.63$.									
8.87	2.275	94 030	5.162	0.530	15.68	7.51	170 240	5.666	0.5565
10.79	3.395	94 490	5.248	0.536	12.8	4.97	170 510	5.580	0.551
12.19	4.424	95 180	5.304	0.548	9.82	2.89	170 780	5.464	0.5475
14.19	6.025	95 870	5.372	0.546	5.77	0.969	170 780	5.234	0.534
16.8	8.51	96 560	5.450	0.550	30.2	28.45	170 780	5.952	0.566
18.8	10.81	97 710	5.504	0.554	30.22	28.6	170 780	5.953	0.566
20.75	13.13	98 400	5.550	0.556	29.4	27.1	171 320	5.942	0.5665
22.86	16.18	99 320	5.596	0.564	7.66	1.825	219 000	5.465	0.564
29.61	27.83	100 240	5.821	0.572	30.56	28.72	225 000	6.077	0.5585
26.44	22.28	102 620	5.674	0.574	30.66	28.95	229 000	6.087	0.558
27.9	24.36	105 500	5.709	0.565	1.861	0.0805	222 900	4.858	0.4365
30.4	29.28	112 900	5.775	0.571	2.702	0.188	228 100	5.030	0.481
30.35	28.68	128 000	5.829	0.564	3.639	0.352	229 270	5.162	0.4945
29.17	26.66	128 260	5.813	0.567	4.355	0.52	230 100	5.242	0.508
27.96	24.67	129 560	5.800	0.570	4.95	0.685	229 000	5.295	0.517
26.63	22.20	130 340	5.782	0.565	5.50	0.8825	228 550	5.340	0.536
25.0	19.40	128 520	5.747	0.563	6.46	1.22	228 100	5.409	0.5365
23.14	16.59	127 480	5.709	0.562	7.003	1.442	228 100	5.444	0.5395
21.4	14.19	125 400	5.669	0.562	7.70	1.756	227 650	5.483	0.542
30.4	29.01	120 400	5.803	0.566	8.48	2.158	227 650	5.525	0.5475
3.168	0.2515	129 560	4.854	0.469	9.008	2.445	228 100	5.553	0.5495
3.84	0.380	130 340	4.940	0.482	9.47	2.735	229 270	5.578	0.554
4.50	0.5295	129 560	5.005	0.487	10.1	3.11	229 270	5.605	0.555
5.005	0.670	127 740	5.046	0.498	10.87	3.618	229 270	5.637	0.556
5.36	0.774	127 480	5.076	0.5005	11.63	4.06	229 270	5.666	0.548
5.645	0.870	127 480	5.097	0.506	13.6	5.54	230 370	5.736	0.548
6.305	1.10	128 260	5.148	0.513	16.0	7.90	229 540	5.806	0.559
6.84	1.31	129 040	5.186	0.518	18.47	10.6	229 000	5.866	0.5635
7.54	1.61	129 040	5.228	0.523	20.9	13.77	228 550	5.920	0.569
8.18	1.92	129 040	5.264	0.528	23.2	16.80	228 550	5.965	0.5645
8.70	2.167	129 040	5.291	0.528	25.5	20.22	228 550	6.006	0.562
9.21	2.443	129 040	5.316	0.531	27.33	23.1	228 100	6.035	0.5605
9.71	2.745	128 780	5.337	0.535	30.2	27.82	228 100	6.078	0.5545
10.29	3.096	128 520	5.362	0.536	6.7	1.330	227 650	5.423	0.543
10.81	3.450	128 000	5.382	0.5395	10.28	3.207	228 550	5.611	0.552
3.147	0.2464	167 000	4.962	0.466	13.52	5.51	229 540	5.733	0.5505
4.005	0.4146	166 740	5.065	0.482	17.53	9.70	229 270	5.845	0.570
4.815	0.615	166 740	5.145	0.494	20.03	12.76	229 000	5.902	0.572
5.445	0.811	167 000	5.199	0.5075	21.8	15.17	229 270	5.939	0.574
5.71	0.8975	167 000	5.220	0.510	23.7	17.68	229 540	5.976	0.5685
6.04	1.011	168 080	5.246	0.514	27.0	22.82	229 000	6.031	0.566
6.485	1.71	167 270	5.276	0.516	30.4	28.32	229 000	6.082	0.557
7.365	1.541	166 740	5.328	0.5235	2.86	0.2143	227 380	5.053	0.4885
7.825	1.760	166 740	5.355	0.529	4.075	0.451	229 000	5.210	0.5035
8.445	2.063	167 000	5.389	0.532	5.00	0.699	230 100	5.302	0.5165
9.19	2.482	167 000	5.426	0.5385	5.435	0.855	229 540	5.336	0.5335
9.55	2.71	166 740	5.442	0.544	6.105	1.092	228 550	5.386	0.538
10.12	3,05	167 000	5.469	0.544	6.78	1.353	228 100	5.430	0.5405
10.72	3.46	167 000	5.494	0.598	7.56	1.714	228 100	5.478	0.548
14.98	6.79	167 270	5.640	0.551	8.70	2.296	229 000	5.540	0.552
16.85	8.72	167 270	5.690	0.558	9.46	2.760	229 820	5.577	0.559
18.6	10.7	167 270	5.734	0.561	10.23	3.228	229 540	5.612	0.560
20.5	13.28	167 270	5.776	0.5685	11.03	3.772	229 540	5.644	0.561
22.25	15.68	167 540	5.812	0.570	30.45	28,42	236 200	6.098	0.556
23.9	18.39	166 740	5.841	0.578	28.64	25.56	237 640	6.073	0.562
26.0	21.22	166 480	5.877	0.566	27.3	23.1	238 020	6.053	0.5615
27.95	24.38	166 480	5.908	0.564	25.82	20.8	237 640	6.028	0.564
29.6	27.05	166 740	5.934	0.5595	24.1	18.0	237 640	5.998	0.562
30.75	28.99	167 000	5.951	0.557	22.0	15.23	236 880	5.956	0.569
29.21	26.67	170 510	5.938	0.564	19.85	12.46	237 640	5.913	0.5765
28.36	25.41	170 780	5.925	0.570	17.83	10.08	237 640	5.867	0.572
27.17	23.2	170 780	5.906	0.568	15.27	7.225	237 640	5.799	0.562
25.15	20.42	171 050	5.872	0.580	11.68	4.245	237 640	5.683	0.563
23.5	17.39	170 510	5.843	0.568	7.99	2.00	236 880	5.517	0.5665
21.13	14.21	170 240	5.799	0.5665	5.51	0.921	236 880	5.356	0.552
19.03	11.31	170 240	5.750	0.565	......	......	......	.....	.....

TABLE 2.—(*Continued*)

Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	Values of: log R	Values of: log (100 f)	Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	Values of: log R	Values of: log (100 f)
(*h*) Run V: $A = 0.02376$ Square Feet; $D = 0.1739$ Feet; and $\frac{L}{D} = 54.62$.									
10.07	3.761	75 600	5.122	0.640	28.37	31.75	167 000	5.916	0.6675
11.62	5.05	75 600	5.184	0.644	27.10	28.75	167 000	5.896	0.663
12.52	5.91	75 600	5.218	0.6465	25.62	25.71	166 740	5.871	0.663
14.88	8.35	75 800	5.283	0.648	23.8	22.18	166 740	5.839	0.664
17.15	11.20	76 000	5.355	0.652	22.0	19.18	167 000	5.806	0.670
19.10	14.03	76 000	5.401	0.656	19.96	15.83	167 000	5.763	0.670
20.96	17.0	76 000	5.442	0.660	17.84	12.63	167 000	5.714	0.670
23.07	20.83	76 400	5.486	0.664	16.03	10.0	167 000	5.669	0.661
28.14	30.83	76 600	5.572	0.662	14.10	7.66	166 480	5.611	0.657
26.79	27.87	76 800	5.554	0.6605	10.20	4.005	166 220	5.470	0.657
25.20	24.78	77 200	5.530	0.662	11.10	4.708	166 220	5.507	0.654
23.96	22.25	77 400	5.508	0.660	12.21	5.75	166 480	5.548	0.6565
1.241	0.0372	78 000	4.226	0.454	1.241	0.0362	166 480	4.556	0.4425
1.509	0.0583	78 000	4.310	0.4695	2.182	0.1432	167 000	4.802	0.550
2.238	0.1419	79 010	4.488	0.524	3.097	0.3123	169 430	4.960	0.584
3.031	0.276	79 430	4.622	0.549	4.10	0.586	169 430	5.082	0.614
3.95	0.509	79 010	4.735	0.584	5.105	0.929	167 000	5.172	0.623
4.94	0.8315	78 600	4.830	0.604	4.60	0.745	166 220	5.124	0.6175
4.465	0.6805	78 400	4.786	0.604	3.565	0.4325	166 480	5.014	0.602
3.56	0.415	78 600	4.687	0.586	2.762	0.248	166 740	4.904	0.583
2.675	0.2162	78 800	4.564	0.552	1.691	0.0885	166 740	4.690	0.563
1.98	0.1186	79 220	4.436	0.553	5.675	1.172	167 000	5.217	0.632
1.443	0.0643	79 430	4.299	0.560	10.80	4.565	166 740	5.496	0.6635
5.54	1.062	79 430	4.883	0.6105	9.80	3.73	166 740	5.453	0.660
10.48	4.05	78 800	5.157	0.638	8.955	3.08	167 000	5.416	0.656
9.55	3.335	79 010	5.118	0.634	8.125	2.504	167 000	5.373	0.650
8.70	2.73	79 220	5.078	0.628	7.125	1.907	167 000	5.316	0.646
7.755	2.166	79 220	5.028	0.628	6.30	1.453	167 000	5.270	0.6355
6.88	1.69	79 220	4.976	0.624	28.3	31.64	222 900	6.040	0.668
6.08	1.295	79 430	4.924	0.616	27.22	29.0	222 900	6.024	0.6625
28.2	31.59	128 000	5.798	0.669	25.8	26.23	222 900	6.000	0.666
26.85	28.3	128 000	5.776	0.664	24.05	23.73	222 900	5.969	0.658
25.4	25.19	128 000	5.752	0.662	22.1	19.4	222 900	5.933	0.671
23.3	21.1	128 000	5.715	0.660	20.2	16.28	222 900	5.894	0.6715
21.48	18.09	128 000	5.680	0.664	18.23	13.2	222 900	5.850	0.670
19.52	14.91	128 000	5.638	0.664	16.08	10.27	222 900	5.795	0.670
17.6	11.97	128 000	5.594	0.658	13.85	7.445	222 900	5.730	0.660
15.72	9.46	128 000	5.544	0.654	10.28	4.23	222 900	5.601	0.6725
13.47	6.97	128 000	5.477	0.656	11.37	5.07	222 900	5.644	0.664
11.92	5.407	128 000	5.422	0.6515	12.2	5.845	222 900	5.675	0.6645
11.09	4.65	128 000	5.392	0.649	10.42	4.29	222 900	5.607	0.666
10.28	4.00	128 000	5.359	0.649	9.655	3.673	223 560	5.574	0.667
1.158	0.0306	130 080	4.418	0.431	9.04	3.182	223 920	5.546	0.661
2.38	0.171	131 120	4.735	0.5505	8.250	2.636	223 920	5.506	0.6595
3.10	0.3115	133 200	4.856	0.581	7.325	2.05	223 560	5.454	0.654
3.82	0.4905	132 680	4.944	0.598	6.535	1.612	223 200	5.405	0.6495
4.95	0.8625	130 080	5.048	0.618	5.92	1.327	222 900	5.362	0.650
4.44	0.683	129 560	5.000	0.610	5.405	1.10	222 900	5.322	0.648
3.60	0.4375	129 560	4.910	0.599	4.395	0.706	222 900	5.232	0.6325
2.70	0.2322	129 300	4.784	0.574	3.421	0.4195	222 900	5.123	0.626
2.024	0.1291	129 300	4.659	0.570	2.703	0.2475	222 900	5.021	0.600
5.50	1.065	129 040	5.092	0.6175	1.831	0.1101	222 900	4.852	0.588
10.81	4.49	127 740	5.380	0.655	1.211	0.0458	222 900	4.672	0.5655
9.92	3.76	127 740	5.342	0.652	1.372	0.0618	222 900	4.726	0.586
9.10	3.124	128 000	5.306	0.648	2.424	0.191	222 900	4.974	0.584
8.07	2.428	128 000	5.254	0.642	3.235	0.364	223 560	5.100	0.612
7.045	1.819	128 260	5.196	0.635	2.981	0.5655	223 560	5.190	0.6235
6.39	1.431	128 260	5.154	0.616	4.945	0.8995	222 900	5.283	0.6365
(*i*) Run VI: $A = 0.02392$ Square Feet; $D = 0.1745$ Feet; and $\frac{L}{D} = 54.44$.									
28.4	31.4	78 600	5.591	0.663	17.56	11.82	83 630	5.408	0.656
26.76	28.1	79 220	5.568	0.667	15.85	9.67	84 260	5.368	0.659
25.8	26.08	80 270	5.558	0.667	13.65	7.15	84 680	5.304	0.657
23.76	21.98	80 690	5.524	0.664	11.84	5.34	84 890	5.244	0.653
21.6	18.02	81 540	5.488	0.661	10.11	3.84	85 310	5.178	0.6475
20.38	15.88	82 160	5.466	0.656	8.51	2.66	85 730	5.105	0.638
18.8	13.6	82 790	5.434	0.658	5.72	1.11	86 150	4.934	0.604

TABLE 2.—(*Continued*)

Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of:			Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	Values of:		
		$\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	log R	log (100 f)			$\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	log R	log (100 f)
(*i*) Run VI: $A = 0.02392$ Square Feet; $D = 0.1745$ Feet; and $\frac{L}{D} = 54.44$. (*Continued*)									
1.348	0.0532	81 320	4.282	0.540	14.91	8.66	166 220	5.637	0.6635
1.943	0.0880	83 000	4.450	0.440	13.09	6.61	166 740	5.580	0.660
10.67	4.198	82 370	5.186	0.640	11.68	5.28	167 540	5.533	0.6615
10.0	3.69	82 580	5.158	0.640	10.57	4.255	167 270	5.490	0.654
9.30	3.158	82 580	5.126	0.6345	12.26	5.82	167 000	5.553	0.6615
8.70	2.73	82 370	5.096	0.630	11.0	4.74	168 080	5.509	0.666
7.89	2.233	82 370	5.054	0.628	10.34	4.17	165 700	5.476	0.664
6.945	1.71	82 370	4.999	0.622	9.60	3.59	165 700	5.444	0.663
5.995	1.238	82 580	4.936	0.610	8.80	2.983	166 220	5.407	0.6585
5.243	0.937	82 370	4.876	0.606	7.90	2.378	166 480	5.382	0.654
4.876	0.808	82 370	4.846	0.6045	6.98	1.836	166 740	5.307	0.6495
4.357	0.6445	82 370	4.797	0.604	5.855	1.269	166 740	5.232	0.6415
4.005	0.539	82 370	4.760	0.599	5.03	0.892	166 480	5.186	0.621
3.66	0.445	82 580	4.722	0.594	4.495	0.710	165 960	5.114	0.6195
3.268	0.3465	82 580	4.673	0.584	3.76	0.492	165 700	5.037	0.6145
2.855	0.2603	82 580	4.614	0.577	3.11	0.325	165 960	4.955	0.599
2.402	0.1835	82 790	4.541	0.575	2.595	0.2203	166 480	4.898	0.588
1.86	0.1151	82 790	4.430	0.5955	2.077	0.1428	166 740	4.780	0.592
4.06	0.5235	128 260	4.959	0.574	28.6	32.6	223 920	6.048	0.674
28.35	32.18	127 740	5.800	0.6755	27.3	29.83	223 920	6.028	0.675
26.46	27.78	128 000	5.772	0.670	26.24	27.62	223 920	6.011	0.6755
24.6	23.74	128,520	5.742	0.667	25.36	25.77	223 920	5.996	0.676
23.01	20.8	128 000	5.712	0.6665	24.36	23.93	224 280	5.980	0.6785
21.6	18.35	128 000	5.684	0.668	23.15	21.62	224 280	5.958	0.6795
20.13	15.92	128 260	5.654	0.668	21.95	19.42	224 280	5.934	0.6785
18.23	13.11	128 780	5.612	0.669	20.42	16.92	224 280	5.904	0.6815
16.03	9.94	128 260	5.556	0.660	19.1	14.71	224 100	5.874	0.6785
14.4	7.95	128 000	5.508	0.656	17.6	12.5	224 100	5.838	0.6785
12.23	5.795	128 000	5.437	0.660	15.77	9.90	224 280	5.791	9.673
3.139	0.325	128 260	4.846	0.591	13.84	7.615	224 100	5.734	0.6725
10.89	4.56	126 960	5.383	0.659	12.33	5.995	224 100	5.684	0.668
10.59	4.32	126 700	5.368	0.6595	11.05	4.84	223 920	5.636	0.6705
10.02	3.85	127 480	5.348	0.656	10.6	4.45	224 100	5.618	0.6725
9.395	3.363	128 000	5.322	0.654	9.90	3.886	224 460	5.589	0.6715
8.66	2.838	128 000	5.288	0.6505	9.28	3.395	224 460	5.560	0.6685
7.85	2.318	127 480	5.242	0.648	8.56	2.862	224 460	5.526	0.665
7.085	1.842	127 480	5.198	0.638	7.62	2.252	224 460	5.476	0.662
6.20	1.381	127 480	5.140	0.629	6.66	1.692	224 460	5.417	0.655
5.406	1.028	127 740	5.080	0.6195	5.85	1.30	224 280	5.361	0.652
4.66	0.746	128 000	5.018	0.610	5.05	0.946	224 280	5.297	0.642
2.36	0.1827	128 520	4.724	0.5885	4.37	0.695	224 280	5.234	0.6335
28.53	32.45	167 270	5.921	0.674	3.598	0.464	224 280	5.150	0.627
27.3	29.8	127 270	5.902	0.6745	2.74	0.2535	224 280	5.032	0.602
26.18	27.24	167 000	5.883	0.673	2.103	0.1499	224 280	4.916	0.6025
25.0	24.77	167 000	5.862	0.6705	14.1	7.805	225 360	5.744	0.6665
23.8	22.5	167 000	5.841	0.672	12.75	6.37	222 900	5.696	0.666
21.85	19.05	167 000	5.804	0.6735	11.08	4.85	221 400	5.632	0.670
20.37	16.60	167 270	5.774	0.676	9.94	3.94	222 300	5.5860	0.6735
18.4	13.45	167 000	5.730	0.6725	8.80	3.075	222 900	5.534	0.671
16.85	11.20	166 740	5.690	0.6695	7.98	2.53	223 200	5.493	0.672
14.96	8.795	166 480	5.660	0.668	......	......	....	......	......
(*j*) Run VII: $A = 0.02418$ Square Feet; $D = 0.1754$ Feet; and $\frac{L}{D} = 54.145$.									
10.54	5.956	75 400	5.184	0.803	2.12	0.192	79 850	4.473	0.706
9.63	5.00	75 600	5.107	0.806	2.983	0.4203	80 270	4.624	0.748
8.825	4.22	76 000	5.070	0.806	3.917	0.7375	79 640	4.738	0.756
7.97	3.423	76 200	5.027	0.806	4.31	0.908	79 220	4.778	0.763
7.11	2.597	76 200	4.978	0.7855	24.6	34.2	74 400	5.506	0.827
6.158	1.928	76 600	4.918	0.781	23.46	30.73	74 800	5.470	0.8205
5.476	1.519	76 800	4.868	0.779	22.0	27.04	75 200	5.463	0.822
5.05	1.289	77 000	4.834	0.778	20.43	23.37	75 400	5.432	0.821
4.38	0.9595	77 400	4.774	0.7725	18.57	19.26	75 400	5.390	0.822
3.46	0.5705	77 800	4.674	0.7525	16.71	15.41	75 800	5.347	0.8165
2.648	0.3197	78 000	4.560	0.733	14.8	12.11	76 200	5.297	0.8175
1.783	0.1421	78 200	4.388	0.726	12.85	9.095	76 200	5.235	0.8145
1.347	0.07995	78 400	4.268	0.718	11.26	6.98	76 600	5.180	0.815
1.57	0.1026	79 220	4.339	0.694	9.69	5.095	76 800	5.116	0.810

TABLE 2.—(*Continued*)

Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	VALUES OF: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	log R	log (100 f)	Mean velocity, V, in feet per second	Loss of head, H_f, due to friction, in feet	VALUES OF: $\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	log R	log (100 f)
(j) RUN VII: $A = 0.02418$ SQUARE FEET; $D = 0.1754$ FEET; AND $\frac{L}{D} = 54.145$. (*Continued*)									
24.59	34.61	127 480	5.740	0.833	8.895	4.36	163 880	5.408	0.816
23.24	30.82	127 480	5.720	0.830	7.875	3.38	164 660	5.357	0.8115
21.82	26.82	128 000	5.691	0.8295	6.783	2.505	165 700	5.295	0.812
20.23	23.36	128 000	5.658	0.830	5.76	1.779	166 480	5.227	0.8035
18.69	19.69	127 480	5.622	0.826	5.105	1.371	167 000	5.175	0.796
16.97	16.16	127 740	5.580	0.824	4.295	0.959	166 480	5.099	0.790
15.09	12.64	128 000	5.530	0.820	3.33	0.5605	166 480	4.988	0.778
13.03	9.45	128 000	5.468	0.820	2.369	0.2735	166 480	4.840	0.762
11.7	7.58	128 000	5.420	0.8175	1.697	0.1331	167 000	4.696	0.740
9.93	5.50	128 000	5.354	0.821	2.876	0.4155	167 000	4.925	0.776
10.46	6.05	131 900	5.384	0.818	3.815	0.7465	167 000	5.048	0.784
9.6	5.088	130 600	5.343	0.816	10.1	5.755	222 600	5.596	0.826
8.75	4.195	129 560	5.298	0.812	9.26	4.82	223 200	5.560	0.8245
7.81	3.33	129 040	5.246	0.812	8.41	3.942	223 200	5.518	0.821
6.76	2.441	129 040	5.184	0.802	7.325	2.95	223 200	5.458	0.815
5.76	1.753	128 780	5.115	0.798	6.41	2.238	222 900	5.400	0.8115
4.895	1.257	128 520	5.043	0.7945	5.55	1.675	222 900	5.337	0.8105
4.22	0.8975	128 000	4.977	0.7785	4.96	1.33	222 900	5.288	0.808
2.966	0.429	128 000	4.824	0.763	4.335	1.01	223 200	5.230	0.8045
1.571	0.1076	128 000	4.548	0.714	3.08	0.505	223 560	5.082	0.801
2.321	0.2486	128 000	4.718	0.739	1.633	0.1348	223 560	4.807	0.7795
3.683	0.685	128 780	4.920	0.7775	2.42	0.3003	223 560	4.978	0.7855
24.44	34.62	167 000	5.856	0.830	3.75	0.748	222 600	5.166	0.800
23.22	31.59	167 000	5.833	0.842	24.5	34.97	222 900	5.982	0.840
21.59	27.2	167 000	5.801	0.841	23.62	32.87	222 900	5.966	0.8445
20.02	23.39	167 000	5.768	0.840	22.0	28.73	222 900	5.935	0.849
18.53	19.82	167 000	5.735	0.836	20.4	24.91	222 900	5.902	0.852
17.05	16.60	167 000	5.698	0.8315	18.88	21.13	222 900	5.868	0.848
15.44	13.61	167 000	5.656	0.8315	17.3	17.60	222 900	5.831	0.844
13.52	10.23	167 000	5.598	0.822	15.51	14.02	222 900	5.784	0.841
11.81	7.82	167 000	5.540	0.822	13.83	10.98	222 900	5.734	0.834
10.71	6.357	167 000	5.497	0.818	12.21	8.50	222 900	5.680	0.830
10.36	6.00	168 890	5.488	0.822	10.68	6.46	222 900	5.621	0.827
9.62	5.138	164 400	5.444	0.8185	......	....	....	.....	......

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

STABILIZING CONSTRUCTED MASONRY DAMS BY MEANS OF CEMENT INJECTIONS

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Synopsis

The title of this paper carries the implication of many difficulties, which to a gratifying degree were surmounted in the process described.

In scope the work embraced three principal gravity dams of rubble masonry, faced with ashlar, aggregating 14 000 ft in length and ranging from 30 ft to 190 ft in height. These structures, situated in the Western Ghauts of India, about sixty miles inland from Bombay, at 2 200 ft above sea level, have been in use since about 1917 for the storage of water for hydro-electric development under the available head of 1 700 ft. In recent years the increase of seepage through and under the dams gave rise to some apprehension as to their continued stability, and the remedy of cement injections was prescribed by a committee of consulting engineers.

Accordingly, the paper describes the methods and results of the borings and the injection of 64 000 bbl of Portland cement into 380 000 lin ft of drill holes in the three dams, working under full reservoir conditions. Incidentally, observation of the process during $2\frac{1}{2}$ yr seems pertinent to the currently moot questions of gravity dam design, particularly as to the uplift and sliding influences, and as to reliance upon the usual assumptions of a monolithic structure.

General Conditions

These reservoirs, all within a narrow zone of 20 miles along the crest of the Ghauts, are situated near the head of drainage areas which develop streams flowing eastward across India into the Bay of Bengal. The dams are the means of intercepting this drainage and storing the catchment from the annual rainfall of from 150 to 250 in. precipitated during the monsoon of June to October each year. The reservoirs have a carry-over capacity for

Note.—Discussion on this paper will be closed in May, 1935, *Proceedings*.

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equalizing any ordinary "failure of the monsoon." The water is diverted westward through tunnels and penstocks to two power stations at the foot of the escarpment where the operating head of about 1 700 ft is utilized through impulse turbines for electric generation of power supplying Bombay and vicinity.

The climate is tropical and frostless, with four months of wet weather and eight months of dry weather, each in superlative degree. The country rock is basaltic, commonly a hard, heavy, dense, blue-gray lava formation, weathering to black in the cliffs, with relatively few fissures or partings, but frequently having a top crust of porous and unsound formation. The top-soil of the district is of volcanic origin, red in color, and of clayey texture; it burns to inferior brick, occurring in irregular pockets of the hills, as alluvium along the streams, or in thin deposits over the extensively bare outcrop of the country lava.

All the dams are constructed of this native rock, quarried and shaped at the several sites. The foundation rock is of the same character, with any unsound top crust presumably removed, although it is suspected that this lacked thoroughness in some instances. In general, the bed-rock and the rock component of all the masonry may be considered good, ranging from reasonably good in the lowest dam, Walwhan, to very good in the longest, Shirawta, and exceptionally good in the highest dam, Thokerwadi.

The other elements—mortar and workmanship—were less satisfactory. The mortar, which is peculiar to Indian construction (known as "surkhi mortar"), was prepared according to custom, wherein: (*a*) The local top-soil of the best clay characteristics is moulded and burned into a crude brick: (*b*) the brick is pulverized and mixed with a due proportion of a grade of lime which is presumed to have certain hydraulic properties; (*c*) to Mixture (*b*) is added the desired proportion of sand (which in these dams consists of the pulverized rock of the sites); and (*d*) the complete mortar ingredients are then ground together, with water, for at least 20 min. The grinding (Step (*d*)) is done in a kind of pug-mill which may be a stone wheel drawn through a circular trough by bullocks, or, on large-scale operations, may be a motor-driven heavy wheel revolving in a steel trough containing the mortar mixture. In any case the resulting mortar is handled subsequently in the usual manner; it sets slowly, ultimately attaining strength and hardness about equal to good lime mortar in brick masonry. Furthermore, the surkhi mortar has, in some degree, the qualities of hydraulic cement which promote its setting and hardening under water.

The Hindu masons are adept with their crude tools at cutting and shaping the small ashlar facing stones, and the stone setting to the neat lines of the structure is also well performed; but no derricks were used for stone setting on these dams, and both the facing and the rubble in the "hearting" were mostly of "one man size," lifted or rolled into position. Therefore, it may be inferred that the stones were imperfectly bedded in the mortar, the joints incompletely filled, and the bonding was insecure. In brief, it was not a high-grade job of hydraulic masonry, although fairly well founded on excellent bed-rock.

The resulting seepage through the myriads of porous joints tended to increase, with some evidences of leaching or erosion of mortar. Hence, the problem arose of sealing the joints, fissures, and voids in a manner best calculated to solidify the structure and stop the leaks. The objectives set up were: (1) Solidification, with assurance of continued stability of the dams; and (2) the reduction of leakage to the greatest extent practicable. It was required that all work be done under full reservoir conditions, with uninterrupted operation of the two power stations (which have a total capacity of 100 000 kw).

Plan of Operation

From the beginning it was recognized that a controlling factor would be the progress of boring. Therefore, ample equipment of several types of drills was provided and a program of use was outlined for the Shirawta Dam. This structure was most urgently in need of attention and the proposed work was most extensive—probably exceeding the work on the other two dams combined.

Electric power was the first requisite and was provided from Company lines through a few miles of branch line terminating in a small sub-station at one end of the dam. A three-phase transmission on T-rail poles at 50 to 100-ft intervals fixed in the top of the dam and clamped to the parapet for the entire length of the dam (7 600 ft), served for the operation of the water pumps, the electrically operated drills, and other small motors.

Air compressors, of the electric-drive, stationary type, in units of about 300 cu ft free-air capacity, were installed conveniently below the dam and served for the operation of from four to eight pneumatic hammer drills and a variable number of the air-driven cementation pumps. Occasionally, air was used for cleaning, testing, and pumping water. A 5-in, water main on the up-stream parapet extended one-half the length of the dam, continuing as a 3-in. line to the far end.

The 5-in. air main was laid along the ground at the toe of the dam for a mile, continuing as a 3-in. pipe the remainder of the way, with 3-in. or 2-in. branches to the top of the dam at intervals of 200 ft or 400 ft. Nine "percussion" (pneumatic hammer) drills were used intermittently. Twelve electrically driven rotary drills, cutting with steel shot, were used for all core drilling.

All the cement pumping equipment was actuated by compressed air, and was built for high pressure, but was used for pressures less than 100 lb per sq in. Four of these horizontal pumps were double-acting with 3-in. plungers; eight others had 2-in. plungers; and all of them had steel ball valves arranged for convenient detachability to facilitate cleaning away encrusted cement at frequent intervals. The valves of all cement pumps, and of the safety valves, were 1½-in. steel balls, steel seated; and they were very satisfactory in operation, with only moderate wear and replacement, or dressing of the removable seat rings.

Electrically driven centrifugal pumps were used for the camp water supply and for maintaining a pressure of 100 lb per sq in. in the water main on the

dam to serve all drilling and cementation operations. Each of the cementation pumps required two mechanical mixing tanks (diameter, 30 in.; and capacity, 10 cu ft) for the various grout mixtures, with platforms for cement handling and with storm shelters during the monsoon season. These units of from two to four cement pumps, with accessories, were installed at intervals of 600 ft along the top of dam, with air and water connections. They were supplied with cement over a narrow-gauge track (2-ft) on steel ties running the length of the dam and on the incline, with power hoist, from cement storage and repair shops below the dam.

A machine shop with the usual equipment, lathe, drill press, shaper, and other motorized tools; a blacksmith shop; drill sharpening outfit, with oil furnace and temperature control for the accurate tempering of steel; electric repair shop; carpenter shop; pipe fitters' shop; a small stone crushing plant for making fine sand; a store house for the large supply of imported fittings and special repair parts (for the upkeep of machines coming from Europe or America), all were needed and provided for making apparatus and for maintaining it in good working order.

A camp for 500—officers, mechanics, and coolies—was constructed, Indian style, of corrugated galvanized-steel sheets on teakwood pole frames. The floors were of concrete and the roofs were covered with grass for heat insulation.

The contract for this work was awarded to the Francois Cementation Company, Limited, of Doncaster, England, which firm detailed a force of twenty-one engineers and foremen to direct operations and to train the local labor. The work progressed in three daily shifts, first at Shirawta Dam, October, 1931, to May, 1933; then at Walwhan and Thokerwadi Dams, November, 1932, to April, 1934.

Drilling Operations

Spacing of Borings.—A cross-section of Shirawta Dam (Fig. 1) shows the position of the several series of borings. It was of first importance to determine by experiment the spacing of drill holes which would best serve for the diffusion of cement grout throughout the mass of masonry. Preliminary assumptions were that the dam was so porous that its mass could be "impregnated" with the grout from borings 50 ft apart; or, at least, that a water-tight cement curtain or cut-off might be placed through such borings at 50-ft intervals lengthwise of the dam.

The first 500-ft length of dam was drilled under this assumption, the holes being bored vertically from the top of the dam 5 ft from the water face and penetrating 10 ft into bed-rock foundation. Simultaneously with this series of top holes, a series of toe holes (termed the X and Y-series) was drilled from the ground at the toe of the dam, with the same spacing interval, but staggered between the positions of the top holes. These toe holes were designed to act first as "relief" holes while injecting the opposite top holes under pressure, and, afterward, to be themselves injected if they failed to be filled under pressure from the upper holes.

The plan was attractive, but it was not fully successful, as will be discussed under "Cementation Operations." Consequently, the spacing of primary borings was made $12\frac{1}{2}$ ft, longitudinally, beginning about $5\frac{1}{2}$ ft from the water face at the top of the dam and drilling either vertically or on a 20 on 1 incline down stream to bed-rock. The drilling was then continued from 10 to 40 ft into the bed-rock, according to indications of hardness of rock, fissures, loss of drill water, and suspicions of "ground leaks" that appeared at the toe of the dam. Even with this spacing it was found impossible to inject grout, or water, or to trace the color from fluorescein dye throughout the intervening mass of masonry which would connect with all the leaks or seepage showing in the down-stream face.

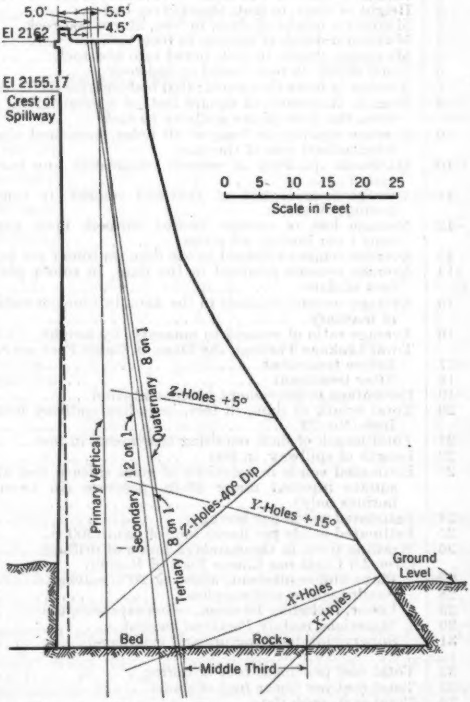

FIG. 1.—TYPICAL SECTION OF SHIRAWTA DAM SHOWING POSITION OF THE SEVERAL SERIES OF BORINGS.

Close Drilling.—As the cementation progressed it was eventually proved that close drilling was indispensable for both the top and the toe series of holes; hence, the spacing for secondary holes was made 6 ft; and, with the following tertiary, quaternary, and special intermediates, the average interval between all holes of all series became 2.1 ft lengthwise of the dam, as shown in Table 1 (Item No. 9) and Table 2. This experience applied only to the systematic location of borings throughout the length of the dam. The location of holes for intercepting and sealing the several large isolated and persistent leaks was a separate problem for each case, requiring special position, direction, depth, and methods of treatment, as hereinafter described.

This drilling experience, with the accessory cementation process, tended to confirm an original opinion that the degree of success in solidifying the dam by cement injections through borings, would be proportioned directly to the number of borings symmetrically placed within the ground plan of the structure.

Definitive Boring.—Innumerable efforts were made to intercept the path of obvious leakage by borings begun directly over the assumed path, or inclined at various angles. These borings were aimed so as to cut the path, or in any manner estimated to provide a conduit for forcing cement grout

TABLE 1.—STATISTICS PERTAINING TO THE THREE DAMS

Item No.	Description	Shirawta Dam	Walwhan Dam	Thokerwadi Dam
1	Length of section*, in feet	....	....	800
2	Height of dam, in feet, above river bed	83	71	190
3	Maximum height of dam, in feet, above bed-rock	132	81	195*
4	Maximum depth of boring, in feet	149	107	230
5	Maximum depth, in feet, bored into bed-rock	45	64	44
6	Usual depth, in feet, bored in bed-rock	10	10	10
7	Number of holes that penetrated bed-rock for 20 ft., or more	29	35	30
8	Area, in thousands of square feet, of a longitudinal section from the crest of the spillway to rock	555.1	275.0	116.0
9	Average spacing, in feet, of all holes, measured along the longitudinal axis of the dam	2.1	1.7	4.0
10	Maximum quantity of cement retained in any boring, in tons †	77.4	16.7	115.45
11	Average consumption of Portland cement, in tons † per boring; all series	1.50	0.77	16.10
12	Average loss of cement vented through open joints, in tons † per boring; all series	0.34	0.05	0.10
13	Average cement retained in the dam, in tons † per boring	1.16	0.72	16.00
14	Average cement retained in the dam, in tons † per linear foot of dam	0.50	0.42	3.94*
15	Average cement retained in the dam, in tons per cubic yard of masonry	0.007	0.008	0.013
16	Average ratio of cement to masonry, by weight	0.004	0.0045	0.0073
	Total Leakage Through the Dam, in Cubic Feet per Second:			
17	Before treatment	22	10	4
18	After treatment	2	1	0.25
19	Percentage improvement by cementation	91	90	94
20	Total length of dam, in feet, including spillway lengths in Item No. 22	7 600	4 472	2 300
21	Total length of dam requiring treatment, in feet	7 600	4 472	1 700
22	Length of spillway, in feet	1 000	1 100	500
23	Estimated solids in solutions of soda silicate and alumina sulfate injected under 80-lb. pressure (in twenty-five borings only)	9.0	....	....
24	Estimated solids per boring	0.36	....	....
25	Estimated solids per linear foot of dam (300 ft.)	0.03	....	....
26	Working time, in thousands of hours of drilling	54.4	29.7	25.2
	Over-All Costs per Linear Foot of Boring:			
27	Plant and equipment, allowing 20% salvage	$0.32	$0.29	$0.91
28	Sundry stores and supplies	0.31	0.21	0.77
29	Labor, including foremen, office expenses, etc	0.83	0.48	1.32
30	Materials, mainly Portland cement	0.33	0.17	0.96
31	Supervision (contractors and engineers)	0.54	0.35	1.20
32	Total cost per linear foot of boring	$2 33	$1 50	$5.16
33	Total cost per linear foot of dams	$63 32	$43 30	$128.50
34	Total cost, each dam	$481 300	$194 800	$219 100

* Middle section, from Station 650 to Station 1 450.
† Long ton = 2 240 lb.

into the fissure, crack, or void that seemed to be the source of the observed leak.

Most of such efforts failed, due to the obscure relation between the outlet, the inlet, and the course of these sluggish types of seepage or small leaks. Effective boring connection with such leaks seemed to be purely fortuitous.

Considering that the first proposed 50-ft spacing of holes involved driving the grout through masonry interstices at least 25 ft lengthwise of the dam, when the cross-wise distance to a free outlet was much less—and that the probable shrinkage cracks would lie cross-wise rather than lengthwise of the structure—there was no logical basis for the hope of constructing a continuous curtain of cement through borings 50 ft apart lengthwise.

It will be understood that this drilling and the accessory cementation were performed for the most part while the reservoir was full, or nearly full, of water. The resulting pressure against the dam maintained the observed

seepage and leaks, and tended to drift the drilling water, as well as the cement grout, toward the land face of the dam. The escape to the water face, through the shorter distance, however, interfered most with the process of cement injection.

TABLE 2.—COMPARABLE CHARACTERISTICS OF OPERATIONS ON THREE DAMS

Description	Shirawta Dam	Walwhan Dam	Thokerwadi Dam		
			North section	South section	Middle section
Specific gravity of trap-rock in foundation and in masonry	2.68	2.60	2.90	2.90	2.90
Weight, in pounds per cubic foot, of composite masonry cores tested	146.0	143.0	160.0	160.0	160.0
Over-all rate of drilling, in feet per hour of drilling: Shot	1.67	2.75	......	......	1.47
Over-all rate of drilling, in feet per hour of drilling: Percussion	5.06	9.14	3.41	3.41	......
Average consumption of steel shot, in pounds per foot bored	1.33	0.76	......	......	1.34
Average spacing of all borings, in feet, lengthwise of the dam	2.1	1.7	5.0	6.0	4.0
Portland cement: Average consumed in tons* per boring	1.50	0.77	0.65	0.35	16.10
Portland cement: Average lost, in tons* per boring	0.34	0.05	0.04	0.02	0.10
Portland cement: Average retained, in tons* per linear foot of dam	0.56	0.42	0.102	0.067	3.94
Portland cement: Average retained, in tons* per cubic yard of dam	0.007	0.008	0.003	0.0017	0.013
Ratio of cement to masonry, by weight	0.0040	0.0045	0.0017	0.0010	0.0070

* The long ton, 2 240 lb.

Character of Drilling.—For training the Hindu drill-runners, work was begun with the air-hammer drills on tripod mounting at the toe of the dam for the *X*-series of holes. As shown in Fig. 1 these holes were projected to intersect bed-rock at the upper and lower limits of the middle third of the base of the dam and then to penetrate the rock a minimum of 10 ft. The *Y*-holes were drilled consecutively with the *X*-holes and aimed upward to penetrate within 6 to 10 ft of the water face of the dam. A year later (spring of 1933), as the finishing touch, the *Z*-holes were drilled, from a traveling stage, at 5-ft spacing, penetrating in the directions shown, to within 6 ft of the water face (see Fig. 1).

Soon after beginning the *X*-holes the hammer-drill runners acquired sufficient skill to undertake the deep vertical holes from the top of the dam. The tripod mounting, set on a timber triangle with steel shoes, to prevent puncturing the pavement, was used most of the time, although a stanchion mounting was made in the local shop and used with good effect in speeding up the changes of drill rods. This device might easily be improved and standardized for use in lieu of the tripod in similar situations.

At first, this top drilling was done in one set-up to a finish at 10 ft in rock; then the "stand-pipe," or grouting connection of 2-in. wrought-iron pipe, 4 to 10 ft long, would be caulked and cemented in the collar of the hole, with a standard pipe coupling at the top. Later, it was found that a more secure setting of the pipe could be made by first drilling the hole only to the depth needed for the stand-pipe, then cementing the pipe firmly in place, coming back 24 hr, or more, later, and drilling through the pipe to the bottom of the hole.

With accurate bit sharpening and gauging it was frequently feasible to drill all the way from the top to the bottom of an 80-ft hole with a $1\frac{7}{8}$-in. bit through the 2-in. pipe collar.

It became general practice to drill through the 2-in. pipe to any depth less than about 130 ft, using the $1\frac{7}{8}$-in. bit with changes at 10-ft rod lengths until bed-rock was reached. Then the bit changes might be needed every few inches of depth and the gauge of the bits would be stepped down to a minimum of $1\frac{1}{2}$ in. This practice was feasible first, because of the relatively easy cutting ground of rubble masonry in surkhi mortar, and, secondly, because of the good drilling equipment and skilled management of the labor. Particular mention should be made of the special type of drill rod provided by the contractor for use with the drifter machines. These rods were of round hollow steel, upset to a diameter of $1\frac{5}{16}$ in. at the ends for connection with square threaded and shouldered nipples of the same outside diameter, which were tightened by rotation opposite the drill rotation.

The rods were supplied in even 10-ft lengths, with a sufficient number of 2-ft and 5-ft lengths, to facilitate changes in depth. Bits of cruciform pattern were found best for the local ground.

With this outfit of rods it was feasible to drill easily to depths of 150 ft with few delays and little breakage or wear of the nipple couplings, which were a controlling feature. In speed, this air-hammer drilling was from two to four times faster than core driling with shot bits in the same ground (see Table 2).

The cutting of both types of drill was faster and easier in this surkhi rubble than would be the case in well made Portland cement concrete of the same stone aggregates. Nevertheless, many of the individual rubble stones, as shown by tested cores, were as heavy, hard, and tough as the bed-rock itself. The average toughness, however, was less and the combination with the rather soft, and brittle mortar made it possible for both types of drill to maintain good clearance and rapid cutting, with only occasional raveling or caving of the hole or jamming of the bit in cracks or fissures.

Core Drilling.—The core drills were excellent, heavy, electrically-driven, machines using rotary shot bits. At first, it was assumed that these drills would be used for all top-hole, deep drilling; but the hammer type of drill "walked away" from the rotary in speed of cutting so that there was a tendency to favor the use of the hammer type. This tendency was disapproved when it came to drilling the thin section of Walwhan Dam, and in all drilling of closely spaced finishing holes where the vibration of the hammer drilling was suspected of shattering the weak masonry, or of interfering with the setting, hardening, and preservation of the films of cement previously injected through adjacent borings.

The progress of the core machines was advanced first by improvements in the rod-handling derricks and in the skill acquired by the operators. Greater improvement was made by substituting smaller bits on these core machines. At first, the borings were made with $4\frac{1}{2}$-in. bits, requiring 5-in. flanged stand-pipes in the collar for grout connection.

This waste of effort and materials was corrected by use of $2\frac{3}{4}$-in. (outside diameter) bits, recovering a 2-in. core and requiring only a standard 3-in. stand-pipe with coupling. No advantage was realized from the borings of larger diameter for cementation purposes. Hollow rods in 10-ft sections, and with excellent couplings, were used. Sludge tubes were used only with the larger bits and core barrels.

To quite an extent the core machines, without derrick, were utilized for starting holes in advance for both types of drilling. The swiveling head on the core drills facilitated starting, or drilling to completion, the inclined holes which became an important feature of the work at Walwhan Dam.

In general, there was good recovery of core for determining the composition and weight of the masonry and the bed-rock. The proportions of rock to surkhi mortar proved to average about 60% rock to 40% mortar. The mortar was firm enough to permit good cores to be recovered although not in as large a proportion as in rock. Diamond drilling was not considered suitable for this ground.

Test Borings.—In the routine of work practically all borings became test holes of a sort: First, in revealing the nature of the local top crust and by indicating the length of stand-pipe needed for reaching solid ground which would not be lifted by the grouting pressure; next, by showing the depth at which partial or total loss of the drilling water would indicate a crack or fissure which must have special attention in the cementation to follow; and, on entering bed-rock, the occasional loss of water (and always the change in cutting speed, the character and color of the sludge, and the general "feel" of the ground) would confirm the height of the dam at that point and develop a profile of the foundation, of which there was no authentic record.

The *X*, *Y*, and *Z* holes served to test the outside shell of masonry and were useful in showing the degree to which the dam was saturated. With full reservoir, the structure was mostly saturated from the bottom to within 2 or 3 ft of the lake level, but at the toe the depth at which the drill would tap water sufficient to fill the boring or to maintain a spouting flow from the "stand-pipe," varied greatly. Such penetration to water from the down-stream face varied from 2 to 20 ft, but commonly it was about 12 ft; the flow seldom increased from beyond that depth, except in the vicinity of bold leaks.

Penetrating the structure from the top, the depth at which drill water would be lost varied from about 30 to 70 ft. Usually, if it occurred at all, it was about 5 ft or 10 ft above bed-rock rather than immediately at the rock contact. This latter experience, applying to all three of the dams, confirmed the judgment that, in general, the dams were well seated in water-tight contact with their foundation rock. The few exceptions to this general rule were in the vicinity of the several serious "ground leaks" appearing at the toe of the dam, as will be further described. The borings from the top were utilized for testing with compressed air, suggesting by bubbles rising in the lake the approximate position, depth, and size of the connecting joints or cracks. In the more open ground the air would show at the joints in the down-stream face.

Next to air the most searching test was a fluorescein dye solution in the water pumped into the boring under the approved grouting pressure—50 to 80 lb per sq in. This vivid green color, derived from a teaspoonful, or less, of the red powder dissolved in a pail of water, would show in face joints or in the boring pipes in from 3 min. to about an hour, and at a distance of from 10 to 100 ft away, according to the size and direction of the connecting passages.

Either or all of these tests were useful in some degree for determining: (*a*) What mixture, pressure, and quantity of cement grout would be required for a given boring; (*b*) whether wastage under water and into the lake was beyond control; (*c*) whether wastage through the down-stream joints or pipes could be stopped by caulking; or (*d*) if the use of a sand mixture might be anticipated.

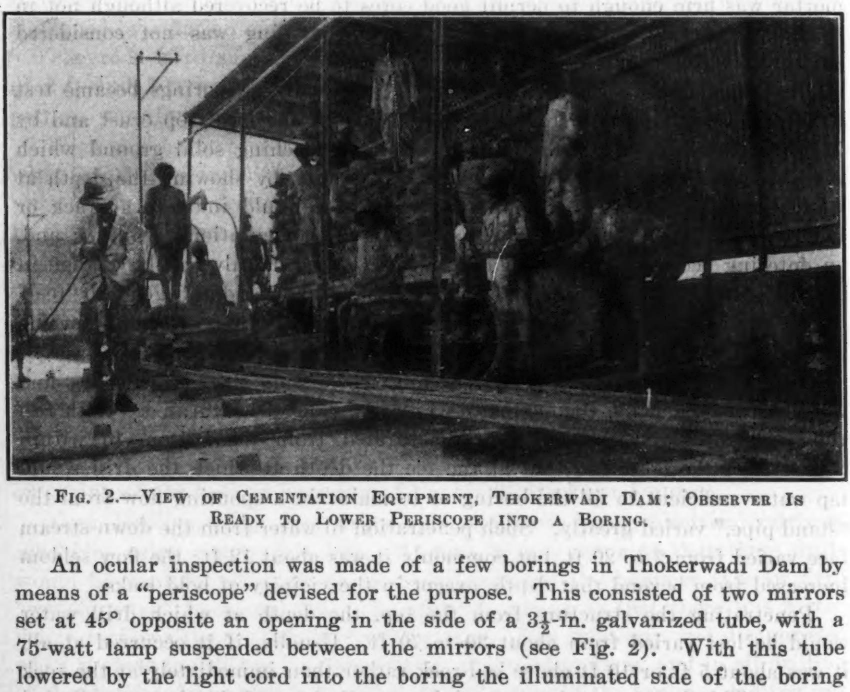

FIG. 2.—VIEW OF CEMENTATION EQUIPMENT, THOKERWADI DAM; OBSERVER IS READY TO LOWER PERISCOPE INTO A BORING.

An ocular inspection was made of a few borings in Thokerwadi Dam by means of a "periscope" devised for the purpose. This consisted of two mirrors set at 45° opposite an opening in the side of a $3\frac{1}{2}$-in. galvanized tube, with a 75-watt lamp suspended between the mirrors (see Fig. 2). With this tube lowered by the light cord into the boring the illuminated side of the boring was reflected to the top by the upper mirror, giving a good picture (the obverse of the core), with a view of cracks and void spaces, down to a depth of about 40 ft for the naked eye and about 70 ft with the aid of binoculars. The device would be more valuable with better equipment and in 6-in., or larger, borings.

In the primary work the drilling got well ahead of the cementation process which, as the work progressed, tended to modify the drilling program and introduced, among other questions, the arguments for and against cementation in "stages" of depth of hole.

Valuable data pertaining to the boring operations at Shirawta Dam, from October, 1931 to May, 1933, may be summarized, as follows:

Total linear feet of percussion boring	172 578
Total linear feet of shot boring	33 951
Total linear feet, all boring	206 529
Shot Used:	
Total, in tons	20.1
Average, in pounds per linear foot of boring	1.33
Average Over-All Penetration Speed, in Feet per Hour:	
Percussion drills	5.06
Shot drills	1.67
Total Consumption of Equipment Materials, in Linear Feet:	
Drill steel	540
Percussion rods	1 700
Shot rods	280
Core barrels	430

The percussion rods used on the work were $1\frac{5}{16}$ by $\frac{7}{8}$ in. in diameter, weighing 3 lb per lin ft, with nipples. The percussion (X) bits were $1\frac{7}{16}$ by $\frac{3}{8}$ in. in diameter, weighing 5 lb per lin ft. The shot rods were $1\frac{3}{4}$ in. in outside diameter, weighing 6 lb per lin ft, with couplings. All threads were square, with a pitch of $\frac{1}{3}$ in. Fig. 3, depicting the borings between Sections 35 and 40 (500 ft) of Shirawta Dam, demonstrates graphically the extent of drilling required. Only the top holes are shown, the X, Y, and Z series of toe holes being omitted.

The Cementation Process

The numerous questions arising in the beginning of operations were resolved successively by experiment as the work progressed. Questions as to the character of cement to be used, its strength and periods of setting, its proper mixture with water, the feasible mixtures with sand, the manner of application, the pressures for safe application under the varying conditions, the sequence of boring and grouting, the control of waste due to cement escaping out of sight under the lake level, or out of the down-stream face joints which might be, or perhaps should not be caulked, the effective handling of the machinery, and the setting up of a system and routine which could be entrusted to the available operatives — all such questions, depending for answer on trial, observation, and discriminating judgment, came up for decision.

Incidentally, critical observation was directed to questions of foundation security, the ascertainable influences of the grouting pressures toward the uplifting effect, with its locality and extent, and to the general question of gravity dam design and some of the assumptions of that science.

Character of Cement.—Since an Indian brand of Portland cement was available, which possessed modern standard qualities, there was little thought of using anything else. There was some debate as to desirable fineness and setting time of the cement for such use, but experience demonstrated that a

straight Portland grade of cement of average setting time has the requisite qualities, particularly as to hardening in place in the presence of an excess of water.

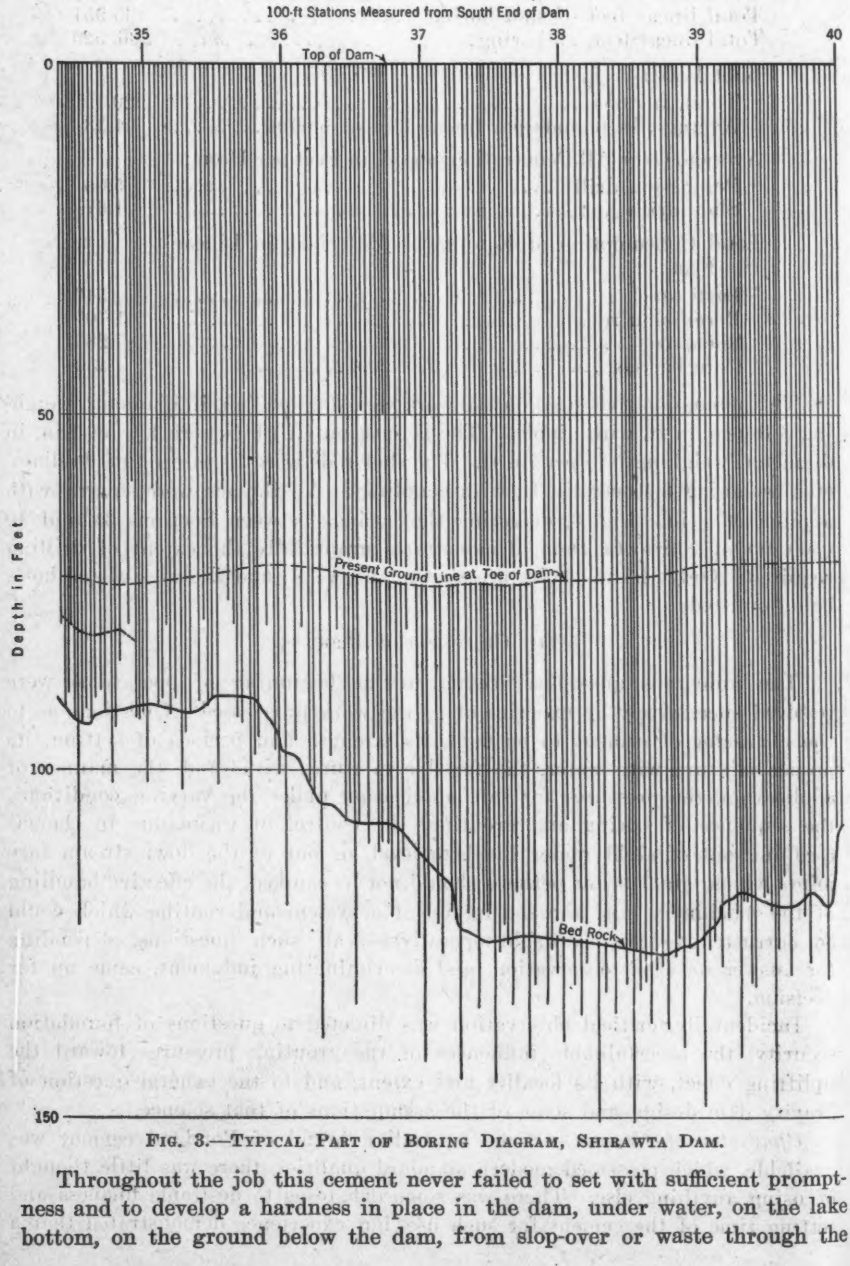

FIG. 3.—TYPICAL PART OF BORING DIAGRAM, SHIRAWTA DAM.

Throughout the job this cement never failed to set with sufficient promptness and to develop a hardness in place in the dam, under water, on the lake bottom, on the ground below the dam, from slop-over or waste through the

joints, and in all crevices where it was injected. The degree of hardness and strength attained was proportional in general to the pressure of placement with accompanying extrusion of water. This was shown clearly by the superior density of cored specimens from the drills at the lower levels where the primary injection pressure had been at a maximum.

There was no conflict with the cement-water ratio law of strength, and it was understood that the excessive use of the water vehicle for grouting involved a sacrifice of strength; but strength of cement in this work is not the prime requisite. The object is to stop the cracks and fill the voids with the heaviest durable material that is feasible for delivery and solidification in place.

Toward the last, in attempting to stop the residual seepage where the regular process had not been fully effective, the standard cement was put through a bolting screen of 150-mesh wire cloth in a trial run. There was a barely perceptible benefit from this removal of all trash, lumps, and coarser bits of clinker, and it seems probable that for cement injections into tight ground some better efficiency may be attained by the use of extra finely ground and resifted cement—with due precautions against its flash setting.

Grouting Mixtures.—The question of widest discussion and experiment concerned the suitable mixtures of water and cement. Proportions were specified by percentages of weight; but for convenience of handling both the water and the cement were measured in bulk when combining them in the mixing tanks.

Cement was purchased by the long ton (2 240 lb) and delivered in jute bags of 1-cwt (112-lb) content. Approved mixtures ranged from 1% (1 part cement to 100 parts water, by weight), in tight ground, to as much as 150% ($1\frac{1}{2}$ parts cement to 1 part water) in open or leaky ground of masonry. The practice was to start all holes with a thin mix and progress to the heavier mixtures as the widely varying conditions demanded.

Ordinary practice in primary holes in Shirawta Dam required a starting mixture of 4 per cent. At Thokerwadi, in the more open masonry of the middle section, the holes were started with an 8% mix and propressed to one of 100% (1 part cement to 1 part water) after a few hours, according to indications of pressure, or the showing of cement color in face-joints or in weep-holes. In the final series of holes in all dams the thinnest mixture (1%), would frequently be rejected at first, but, in general, these final tight holes would run for several hours taking 1% or 2% mixtures under maximum permissible pressures of 70 to 80 lb per sq in.

In none of the dams was the mass of the masonry sufficiently open to permit mixture even of the finest sand in the grout. The ordinary porosity or intersticial spaces of the masonry were not interconnected or large enough to accept any solids coarser than the cement flour in suspension, and, frequently, even clean water dyed with fluorescein would not penetrate to a showing of color at a distance of more than a few feet from the hole under the pumping pressure.

The large use of cement at Shirawta Dam was in the numerous definite cracks, joints, leaks, and relief holes through and under the dam, and it was in this type of structure that the heavy mixtures of cement were required in combination with the prepared sand, graded through a 20-mesh sieve. The further complication of leakage and wastage of grout into the lake, through the masonry joints connecting with the holes under the injection pressure, at times made the sand mixtures imperative.

Methods of Application.—It was originally proposed to install the cement pumps in two batteries, each of six pumps at each end of Shirawta Dam, $1\frac{1}{2}$ miles apart, for delivery of grout through $1\frac{1}{2}$-in. or 1-in. pipes to all borings in the top and toe of the dam.

The actual beginning was made with a group of four pumps set at the cement shed near the toe of the dam 80 ft below the top, with grout delivery to the top and toe holes, through 1-in. pipes under pressure of 70 lb at the top of the dam (see Fig. 4).

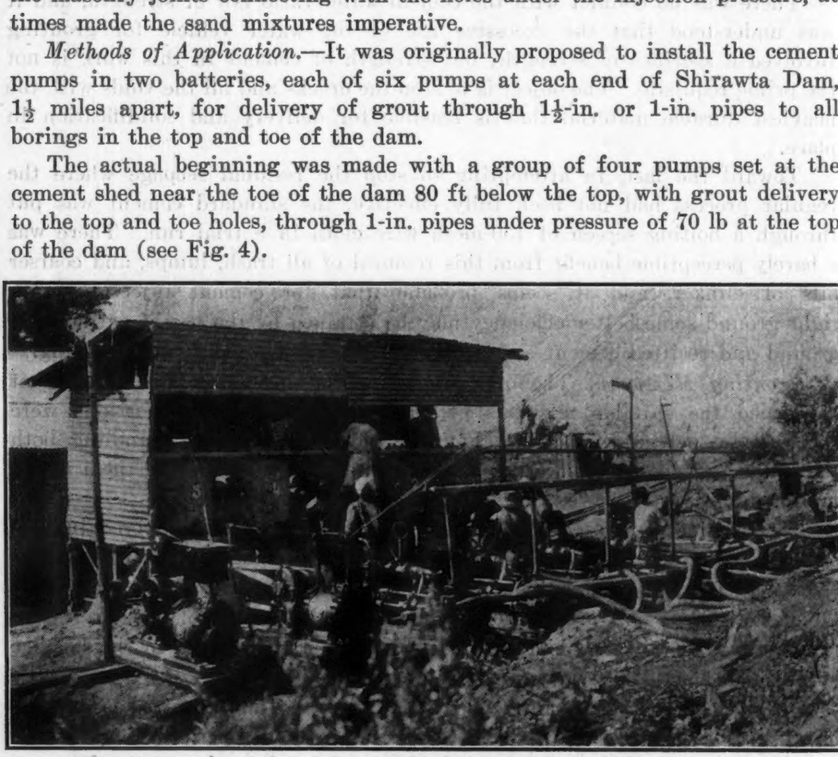

FIG. 4.—SHIRÂWTA DAM; THE FIRST PUMPING PLANT, NEAR TOE OF DAM.

It was soon realized, however, that difficulties would be encountered in regulating the pressure in the borings, and that incrustation of cement in pipes and pumps was a factor that militated against distant and indirect delivery. Hence, the plan was adopted for setting the pumping plants at 600-ft intervals in tandem position against the parapet, occupying about one-half the 12-ft top width of the dam, leaving room for drilling and transport operations. This plan was applied similarly to the other dams, giving close regulation of all pressure grouting, whether in the top or in the toe borings. Portability of these cementation plants was required for the successive series of borings in the same ground, and this was readily achieved by shifting the several units of pumps, tanks, stages, and accessories on the small flat-cars used for cement transport on the narrow-gauge track—the water, air, and electric connections being ready for repeated use at each 600-ft station.

Each such plant consisted of two 3-in. plunger pumps and two 2-in. plunger pumps set at opposite ends of a double stage, 4 ft wide, 4 ft high, and 16 ft long, on which the cement in bags was stocked for mixing first through a 90-gal tank on the stage and then through a similar tank on the floor used for the pump suction chamber.

The cement stage and tanks were enclosed during the monsoon. Each tank, 30 in. in diameter, was equipped with a stirring paddle at two heights on a vertical shaft and geared to a countershaft with sprocket connections for operating the upper and lower tanks together from an electric engine under the stage. The prescribed portions of cement were measured in boxes and dumped through a coarse screen into the upper tank. The paddles being continuously rotated, served for good preliminary mixing in this tank, which was discharged into the lower tank through a screen of $\frac{1}{8}$-in. perforations made tight around the edges to prevent any lumpy or fibrous material from entering the pump suction.

The grout from the pumps was delivered to the borings through $1\frac{1}{2}$-in. wrought-iron screw pipe for the 3-in. pumps and 1-in. pipe for the 2-in. pumps, each pump having individual suction and delivery connections to avoid the clogging and stoppage incident to interconnection with other pumps. Flexible connection with suitable hose and couplings was provided both at the pump and the boring ends of the grout ranges. At each boring connection with the stand-pipe in the hole there was placed a lever type of safety valve with adjustable weight for blow-off at 70 lb, or other prescribed, pressure. This safety-valve connection was also applied at the top of the dam to the toe holes, which likewise were treated from the pumps on top of the dam.

Other fittings of the boring intake included a stop-cock on the stand-pipe above which a 4-way fitting provided the grout intake on one side. A blow-off or relief valve was placed on the opposite side, with the safety valve above or on a "tee" of the grout intake, while provision for feeding sand into the grout stream was made in the top of the 4-way cross.

A sanded mixture passing through the pumps was inadvisable for obvious reasons, so other methods were devised for introducing sand to the limited extent that was found to be beneficial. When sand was used it was practically all in connection with stopping the leakage of grout into the lake from the top holes, or wastage into ground leaks from the toe holes. In either case the sand could be run in a small continuous stream, or in handfuls, through the open-top 4-way fitting into the grout stream from the pump—falling freely into the top boring or into the down-pipe delivering to a toe boring. This process might continue for several hours until the gravity head would build up to the overflow level at the fitting, when the sand dosing would be stopped and the pump pressure applied to the neat grout stream only.

In some of the worst leaks to the lake a method was devised for delivering sanded mixtures by gravity head from two of the regular mixing tanks set on a trestle 10 ft above the top of dam. This proved quite satisfactory within short ranges of delivery pipe, but was not effective for general use (because of the clogging of the pipes and the fittings) and was abandoned in favor of the method first described. In some of the larger leaks, furthermore,

there was resort to a single-drum pneumatic grouting machine, which happened to be available, but under all the local conditions this method was not as flexible, controllable, and effective as the straight pumping for the neat grout and the combination of the pumping and gravity delivery for the sanded mixtures.

Suitable Injection Pressures.—The degree of pressure safely applicable to this peculiar masonry was a matter for trial and judgment.

It was undesirable forcibly to replace any of the weak surkhi mortar with new and better cement, even had this been practicable in the section of dam as shown, with full lake static pressure against one side, atmospheric pressure on the other, and the injection pump between.

It was not desirable to apply a disrupting pressure in the process of cementation, but it was desirable to take advantage of the most effective pressure for driving the cement-laden water into the smallest passages connected with the borings and for expulsion of the excess water while compacting the cement in all accessible interstices.

With the perfectly controlled pumps, delivering through adjustable safety valves, as mentioned, it was possible (guided by the drilling log of the hole and the previously described tests) to fill the hole with the starting mixture and gradually to build up pressure to about 50 lb, at which there was little danger of any harmful effect.

In a number of early trials it was found that full air-line pressure of 100 lb per sq in. seemed to be harmless, yet, apparently, no more effective for injecting greater weight of cement. Other trials of pressure greater than 75 lb per sq in. produced some signs of distress, such as a slight upheaval of the top pavement, a slight displacement of the facing ashlar masonry, excessive leaks to the lake, or excessive discharge into relief borings. From these trials, and the general sense of caution, the limiting pressure of 70 lb per sq in. was adopted for all three dams with a few special exemptions under personal observation where 80 lb per sq in. could do no harm and might be of some help—but it was never proved that results were any better with pressures greater than 70 lb per sq in.

This regulation of pressure was applied to the normal conditions where borings could be filled with the approved starting mixture under pressure. In the very leaky ground there were many borings that could not be so filled and, therefore, required to be "coaxed" along with varying mixtures, including sand, until it became possible to staunch the waste into the lake or toward the down-stream face or the toe of the dam.

It was in this latter class of borings that the principal difficulties were encountered, particularly in the numerous cases where it was impossible for a time to tell where the grout was going or how to keep it in the wall; and, hence, a tendency was developed to persist for hours, or days on end, in pouring thin grout down a hole, siphoning through the pump and falling freely into the boring.

Sequence of Operations.—From the first it was a question whether to grout the primary holes consecutively or to skip one or more holes so as to isolate the areas under pumping pressure. Furthermore, it was a question

whether to grout the top holes first with the toe holes open for "relief" (if relief were needed), or *vice versa*; and whether to grout the top (vertical) holes in separate stages of depth, from the top down, so as to "seal" the restricted horizontal zones of the dam all the way down to bed-rock before sealing the central and toe areas. It was argued by some that there was danger of inducing uplift at the heel such as to threaten the stability of the structure if the grouting was begun at the toe.

The first section of 600 ft of dam became the laboratory for testing these questions. The conclusions reached were that: (1) Isolated grouting effected no benefits; (2) toe holes appeared to be unnecessary and ineffectual for relief; and (3) the only advantage of "stage" injections was realized when the leakage from the boring was so great that the pump could no longer deliver the grout to fill the hole and build up the pressure.

Conclusion (1).—Isolated grouting proved to be of no benefit; rather, it interfered with economy, and showed some tendency to hinder the flow of grout from subsequent holes. The method of maintaining the forward side of the borings always unobstructed by previous grouting, was deemed most satisfactory.

Conclusion (2).—Toe holes seemed to be more useful for the initial grouting of bed-rock and the bed joint. This plane (the bed joint) was intersected at the third points of the base, as shown in the cross-section, Fig. 1. Incidentally, and without appearing to induce uplift at the base, or other objectionable stresses within the structure, the earlier grouting through these toe holes tended to produce a container for the upper grouting by sealing the base and the lower zone of face-joints. Caulking was indispensable in this matter, making for the retention of cement in the wall, permitting the escape of excess water in the grout, and preventing the excessive waste of cement to the ground below the dam.

Conclusion (3).—The only advantage of "stage" injections was in cases where the leakage from the boring was so excessive that the pump lacked the capacity to deliver the grout fast enough to fill the hole and build up pressure. The great disadvantage was in the delay, expense, and interference with orderly progress. Consequently, the stage system was not employed when pumping capacity was available for initial pressure on the hole. It was adopted systematically in two instances, however; First, at Walwhan Dam where the thin top section (20 ft high) had to be grouted under lower pressure than was required in the base section; and, second, at Thokerwadi Dam (190 ft high) where the upper measures were of concrete and the lower measures of loosely built rubble masonry, requiring different and separate treatment.

Conclusions (1) and (3) were confirmed by all subsequent experience on the three dams. Conclusion (2) was modified by further experience at Shirawta in places where the lower face-joints were relatively tight. It was found expedient, in such places, first, to grout the top holes and thereby in some degree to form a curtain or cut-off in the up-stream portion of the dam from top to bottom. This was the preferred method when the cement could be retained in the dam.

In localities where the leaky fissures and joints permitted excessive waste of cement through passages that could not be caulked, however, it was necessary first to seal such vents by grouting the base and lower measures of the dam through the toe holes. Consequently, the "top-toe" precedence became a question of expediency, with change of sequence to meet local conditions, in which the apprehension of uplift appeared not to be justified by any detectable symptoms.

Induced Pressures.—Observation throughout the work, with occasional testing by pressure gauges, stand-pipes, and stop-cocks, revealed a limited transference or transmission of the pump-injection pressure through the body of the masonry between adjacent holes. There was occasional, or perhaps frequent, interconnection of borings, at from 5 ft to as much as 50 ft apart, through more or less definite cracks or jointed planes. Furthermore, there were many instances in which the thin grout traveled some distance and appeared in face-joints 20 to 75 ft from the borings when under injection pressure.

In many cases with toe holes being injected, the adjacent—or, frequently, the alternate—hole would become filled with grout under nearly undiminished pressure, so that with a stop-cock on the companion hole the diffusion of grout would be completed within the area influenced by the several holes.

In the larger leaks and in the case of ground leaks, on the other hand, interconnections that occurred transferred the injection pressure, or even the low gravity pressure, through a considerable length of duct, if not over an extensive area. In general, however, there was no such spread of pressure along jointing planes from borings being injected as would result in a menace to the stability of the masonry.

Exceptions to this general rule were noted in the special situations where "ground leaks" occurred—at Stations 6.5, 17.0, 30.0, and 34.0, of Shirawta Dam. In these special cases the routine system was modified to include many additional borings and devices for throttling the gushing leaks gradually while aiming to avoid dangerous intensity and areas of applied pressure.

Routine of Cementation.—Usually, holes were drilled well in advance of the grouting so that the latter process, in a given series of borings, could be advanced in the consecutive manner, with the forward breast of the dam as free as possible for the infiltration of grout. The first move would be to wash out the one or more holes to be injected, making sure of the removal of all drill sludge or subsequent ravelings, down to the bottom of every hole, with a tolerance of 1 ft, or 2 ft, where complete clearance was difficult.

There was a tendency for these borings to accumulate sediment after standing some time, even when they had been plugged at the top after the drilling was completed. The drilling log would be studied for indications of loss, or inflow, of water, which study would be supplemented by the air, water, or dye tests as required, all having the effect of cleaning the walls of the boring and flushing out any cracks or fissures that would serve as conduits of grout into the masonry. Usually, a leakage test would be applied by measuring the rate of inflow under gravity head from the top of the dam. This rate varied from 3 to 30 gal per min, or more.

The top fittings, stop-cock, safety valve, blow-off, and the intake for sand, would then be placed, and the grout connection made. One tank of clear water would go through the pump, to expel the air, fill the boring, and, in tight ground, initiate the pressure, although in most ground of primary to tertiary borings the total delivery of the pump up to 30 gal per min would fall down the hole without producing pressure.

The addition of from 1% to 8% of cement to the water at first would sometimes initiate, say, 20 lb of pressure at the top of the dam, and this would be considered a satisfactory working pressure for half an hour, or half a day, according to the "feel" of the ground. There was always the danger of choking the hole and sacrificing the penetration of the finer connecting fissures by too much cement, too much pressure, or too much speed of delivery. Therefore, the vigilant regulation of mixture, pressure, and speed of pump was required continuously.

For safety, the rule was: "When in doubt, continue the thin mix and low pressure." Consequently, the operating foreman tended always to produce a tenuous mix and low pressure, or no pressure. The high record for this cautionary state of mind was the "injection" of one of the primary holes by pumping the 2% to 4% mix to the intake fitting and letting it fall down the hole nearly continuously for three weeks before the grout clogged and became subjected to pressure. Afterward, it was discovered that much of the cement had accumulated and set hard in a flat cone at the bottom of the lake against the dam. A somewhat less flagrant case of caution occurred on the opposite side of the dam in a boring that consumed nearly 100 tons of cement during a week of pumping in a mixture of from 2 to 100 per cent. Most of this cement was accounted for later by trenching in the rock spoils below the dam where a fairly good grade of concrete had been secretly produced by the infiltration method. (See Table 3 for estimated losses of cement.)

Much experimentation was devoted to the determination of the optimum mix, with ideal pressure and rate of delivery. The optimum was never found. The behavior of each hole was a law unto itself. Gradually, however, the judgment was developed that the extremely thin mixture, less than 2%, was not desirable or effective and that initial pressure on each hole was necessary to secure the best effect, the rate of delivery to be adjusted accordingly.

After washing and testing a hole with clear water, therefore, the approved routine was as follows: (1) A more careful measurement and adjustment of the mixture; (2) constant inspection of pumps, pipes, fittings, strainers, etc., for assurance that there was no stoppage along the line and that the indicated pressure was on the boring rather than on some plugged connection; (3) beginning with a 4% mixture, delivered fast enough to fill the boring, and to develop 10-lb pressure at the top within half an hour; (4) holding that pressure as long as any good effect appeared in the form of grout spread in the down-stream face-joints, or otherwise as might be judged; (5) thickening the mix to 8%, 16%, and as much as 100% if no waste was detected in lake soundings or in uncaulked down-stream joints or vents; (6) continuing to adjust the mixture and the pressure to the best balance for

maintaining the injection during a maximum period and for a maximum weight of cement retention in the dam; and, (7) continuing the injection to "refusal" at a pressure of 70 lb per sq in.

TABLE 3.—SUMMARY OF CEMENTATION OPERATIONS

Series	Borings: Number: Total	Borings: Number: Total drilled into bed-rock	Borings: Depths in Linear Feet: Total	Borings: Depths in Linear Feet: Average: Per hole	Borings: Depths in Linear Feet: Average: Depth per hole drilled into rock	Cement Consumed, in Tons (2 240 Pounds): Total	Cement Consumed: Lost	Cement Consumed: Retained
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(a) Thokerwadi Dam; February, 1933 to March, 1934; Middle Section (Station 650 to Station 1 450)								
Primary	70	69	11 136	159	13	2 162	22	2 140
Secondary	65	65	10 596	163	13	794	8	786
Tertiary	30	30	5 194	173	20	122	2	120
Quaternary	18	18	3 297	183	11	33	1	32
Quinary	15	12	2 525	168	6	78	1	77
Total	198	194	32 748	165	14	3 189	34	3 155
(b) Shirawta Dam; October, 1931, to May, 1933.								
Primary	453	363	38 202	84	11	3 040	776	2 264
Secondary	213	64	15 556	73	9	407	47	360
Tertiary	391	190	32 714	83	2	196	29	167
Quaternary	744	451	67 244	90	2	239	34	205
X	689	536	21 717	31	6	1 113	301	812
Y	480	....	12 098	25	....	362	44	318
Z	616	....	16 818	27	....	111	6	105
Chemical	25	25	2 180	87	3	1	....	1
Total	3 611	1 629	206 529	57.2	6	5 469	1 237	4 232
(c) Walwhan Dam; October, 1932, to February, 1934								
Primary	611	605	46 799	77	10	1 220	83	1 137
Secondary	273	272	21 423	78	9	134	7	117
Tertiary	398	166	24 564	62	6	195	10	185
Quaternary	57	42	4 199	74	3	25	4	21
X	431	426	16 415	38	5	174	9	165
Y	423	....	6 884	16	....	44	2	42
Z	353	1	8 361	24	5	67	4	63
Sluice roof	4	3	177	44	7	9	....	9
Gate shafts	.....	....	....	.....	....	16	3	13
Duct line	2	2	44	22	10	23	2	21
Roadway	15	15	1 291	86	76	68	1	67
Total	2 567	1 532	130 157	51	8	1 965	125	1 840

Thoroughness, rather than speed, was always the rule of action; hence, it was not the practice to force the pressure at once to the maximum, but rather to give every opportunity for the most fluid mixtures of cement to penetrate into the smallest and most remote fissures or voids which were accessible from the boring under treatment. As the work advanced through successive series of borings, the masonry became tighter, absorbed less grout per boring, and the flow became more sluggish.

Accordingly, because the pumps and pipes became encrusted with cement, the practice was developed of connecting two, three, or four adjacent borings to a single pump through its 1-in. delivery pipe, thereby speeding the main flow while retaining the desired pressure and obviating the clogging of apertures

and pipes. These multiple connections could be made only with holes of similar absorption; otherwise, a loose hole in the group might take an undue proportion of grout and prevent the other holes from securing their effective pressure and share of the fluid.

Caulking Vents.—An important part of the cementation process was to caulk the face-joints of the masonry in order to retain the cement in the wall. At innumerable joints between the small-sized ashlar facing stones, in both faces of the dam, the pointing mortar had long since disappeared. On the up-stream face, in the narrow zone exposed above lake level, these joints were readily caulked against any escaping cement. Below lake level this was not expedient without the aid of a diver, and it seemed inadvisable to retain a diver for full time at the site to repair the occasional bad leaks under water. Many of the under-water leaks could be stopped by adjusting the cement mixtures and pressures or, in obstinate cases, by dosing the mixture with sand. The introduction of sawdust or rice chaff was suggested for "choking" these troublesome leaks to the lake, but this was discouraged, and only the selected sand was approved as being heavy and imperishable.

The most serious difficulty up stream was to indentify a leak at a great depth under water. The volume and time interval of rising air bubbles would show roughly the extent and position of some of the larger leaks, whereas soundings with a cutting-edge and a 2-in. pipe trap-valve, would detect deposits of cement on the lake bottom below the leak. On the down-stream face the joints were caulked systematically by a gang of about twenty men during the primary grouting, diminishing gradually toward the last, when only occasional joints required such attention.

At first, an effort was made to clear the joints of loose mortar and to insert more or less continuous strands of old rope packing; but it was soon found best to leave all joints open for the free escape of water and to resort to caulking only when the cement that escaped was of the same density, approximately, as the mixture that was being injected.

Sometimes a hole, or a group of holes, would be under pressure for hours before the cement color would show in the extruding water. At other times, the full mix would show almost immediately after starting an injection, with water and cement being expelled over large areas of face-joints opposite and at considerable distances lengthwise of the dam from the positions of the borings.

Equipped with ladders, slings, kits of tools, a supply of hemp rope strands, old bag strips, and wooden wedges, the caulking force was in constant readiness to staunch the waste of cement as soon as the inspector would decide that sufficient water had escaped and it was desirable to hold all cement in the wall and force the water and grout to further penetration. In primary work large quantities of the old rope strand were caulked in tight or held in place against the pressure by wooden wedges driven into the joints. Toward the end (and for the most part, at Walwhan Dam), strips of old cement sacking, caulked lightly in place, were found most convenient and effective in the tighter joints having less residual pressure from the injection.

A notable phase of these caulking operations was the occasional uplifting, or displacment, of the down-stream ashlar, which was poorly bonded into the rubble masonry. In fact, the facing ashlar was a veneer rather than an integral part of the dam. In numerous cases a single ashlar block would be lifted out like a loose tooth, by the grout pressure combined with the surround-

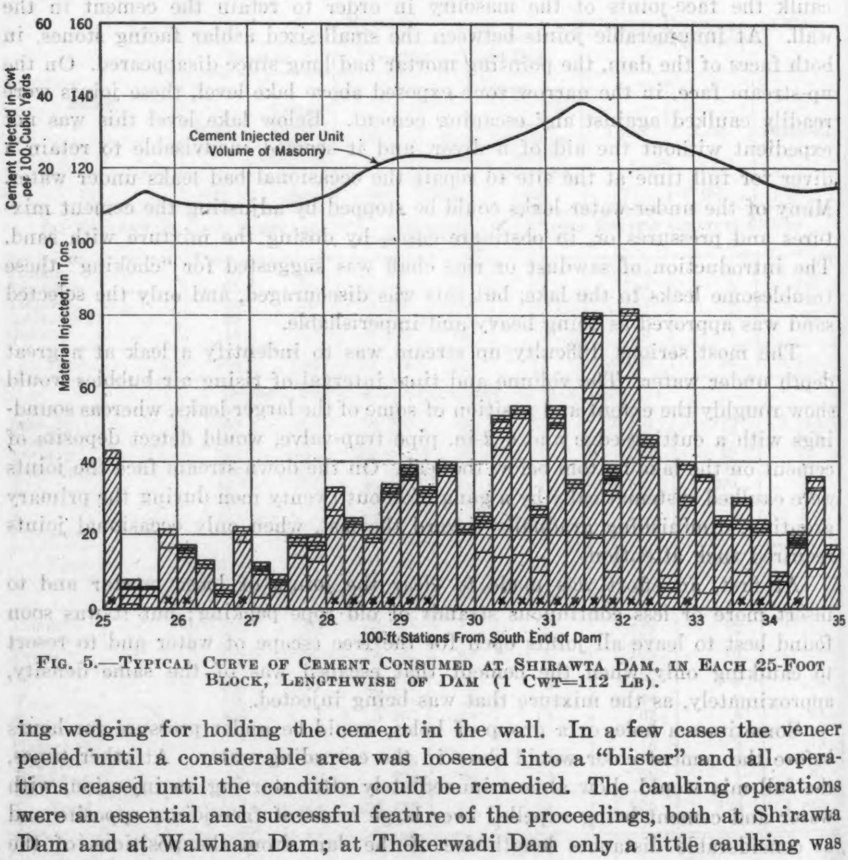

FIG. 5.—TYPICAL CURVE OF CEMENT CONSUMED AT SHIRAWTA DAM, IN EACH 25-FOOT BLOCK, LENGTHWISE OF DAM (1 CWT=112 LB).

ing wedging for holding the cement in the wall. In a few cases the veneer peeled until a considerable area was loosened into a "blister" and all operations ceased until the condition could be remedied. The caulking operations were an essential and successful feature of the proceedings, both at Shirawta Dam and at Walwhan Dam; at Thokerwadi Dam only a little caulking was required.

The cement consumed at Shirawta Dam is illustrated by the typical curves in Fig. 5 (Stations 25 to 35). Sections marked x are those in which reservoir leaks, or ground leaks, caused excessive loss of cement. Figs. 6 and 7 are views of a leak at the toe of Shirawta Dam before and after cementation. Other views are shown in Figs. 8 to 12. Fig. 8 shows some typical bad leaks (including the "big leak") and small spurts before remedies were attempted. Fig. 9 demonstrates a gradual approach to the stopping of the "big leak" as it appeared in May, 1932. In Fig. 10 the ashlar masonry has been removed after a typical blow-out. No disturbance was noted in the interior masonry (see Fig. 11). In this case the pressure on the 6-in. pipes was released and

borings were driven in an effort to intercept the flow along the stratum at the floor of the old inspection tunnel and below it. A view above the site of the big leak, one year after its treatment, is shown in Fig. 12. Note the 6-in. pipes.

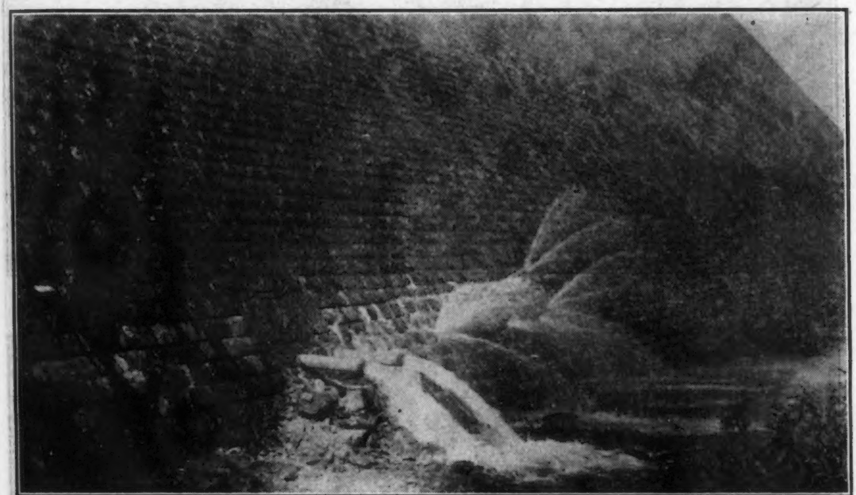

FIG. 6.—SHIRAWTA DAM; LEAKS AT TOE BEFORE CEMENTATION.

FIG. 7.—TOE OF SHIRAWTA DAM AFTER CEMENTATION.

STOPPING THE LARGEST SINGLE LEAK ("BIG LEAK")

The total leakage through Shirawta Dam was 22 cu ft per sec. Of this about 20% appeared in joints and pipes near Station 6.5 from the south end of the dam. At this point an inspection tunnel had been drifted into the dam

FIG. 8.—SHIRAWTA DAM LOOKING NORTH FROM STATION 6, BEFORE BEGINNING WORK.

FIG. 9.—GROUND LEAK BLOCKED BY SAND BAGS AS A GRADUAL APPROACH TO STOPPING "BIG LEAK."

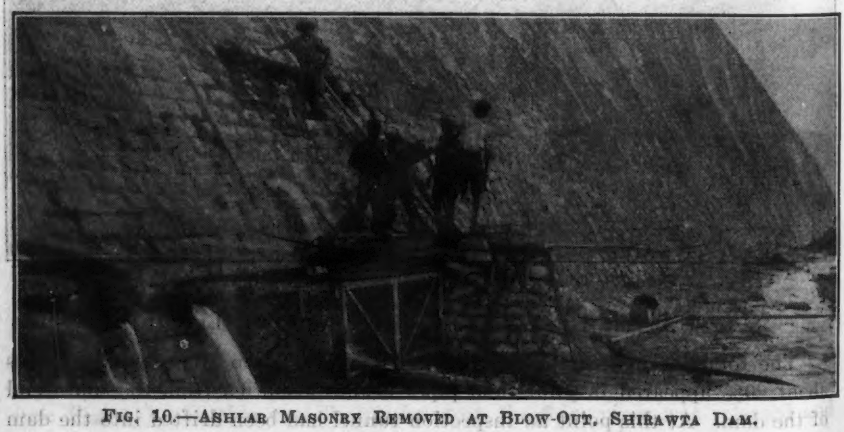

FIG. 10.—ASHLAR MASONRY REMOVED AT BLOW-OUT, SHIRAWTA DAM.

to within about 12 ft from the water-face. This tunnel had been back-filled with cement rubble and drained by two 6-in. pipes and one 4-in. pipe, perforated with 1-in. holes and bedded in the floor.

Fig. 6 shows the leakage (about 5 cu ft per sec) in the pipes and joints of this vicinity before beginning cementation. Fig. 7 shows the condition after completing the work. The final result of work on the entire dam, $1\frac{1}{2}$ miles long, was the reduction of the total leakage from 22 to 2 cu ft per sec (see Tables 1, 2, and 3). The exploratory tunnel had entered a stratum

FIG. 11.—SHIRAWTA DAM, SHOWING FACING STONES REMOVED FROM BLOW-OUT LEAK (NO HARM DONE).

FIG. 12.—SHIRAWTA DAM, SHOWING Z-HOLES BEING DRILLED ABOVE SITE OF "BIG LEAK" ONE YEAR AFTER SUCCESSFUL CEMENTATION.

of masonry 1 ft to 3 ft in height near the base of the dam in which it appeared that the mortar had been particularly inferior and seriously eroded, thereby permitting the heavy concentration of leakage at the site of this drift, or tunnel.

The cementation program throughout the length of the dam included the control and sealing of this most conspicuous leak, as well as several other serious concentrations later discovered at the base of the dam. No sealing medium other than Portland cement grout (pumped under pressure) was used, and the work was done under full reservoir conditions.

Sequence of Attack.—The first move was to proceed with the routine grouting of primary, secondary, and toe holes in the flanking sections of the dam, aiming to solidify the masonry on both sides and beneath the area of gushing leakage. In doing this there was such interconnection through the masonry, between the several borings and the 6-in. pipe drains, that much grout was wasted.

It was not considered safe to close the drains and grout the entire area under the reservoir head as magnified, or influenced indeterminately, by the pumping pressure. The policy, therefore, was to progress from one hole at a time with such adjustment of mixture and pressure as would gradually seal the face-joints, after due caulking, on both flanks of the big leak through the pipes. Narrowing the outlet in this manner resulted in troublesome loss of cement under water in the lake on that side while at the down-stream toe a bad "ground leak" was developed which rose to the surface from the base of the dam or from inaccessible joints a little above the base.

The leaks to the lake were stopped by the sand mixtures. The ground leak was first weighted with sand bags (see Fig. 9), and then intercepted by a number of drill holes suitably grouted under balanced pressure until the boils ceased and the area was solidified for retaining the subsequent injections within the dam. Further attempts to check the main gushers seemed to set up pressures in the relief borings which might become a menace to the stability of the dam, in the somewhat crippled condition of the section which at best was none too strong (see X-section Fig. 1).

At one stage of the proceedings a warning of trouble was given by the outburst of a leak through the down-stream face about 20 ft from the point where injection pressure was assumed to be at a maximum. This leak soon raised a blister under the ashlar veneer (afterward stripped as shown in Fig. 10), whereupon work was stopped pending a new plan of attack. During the pause in operations it was found, by air test and soundings in the lake, that leakage of grout on that side had occurred at a depth of about 70 ft, or a little above the bed of the lake. This suggested the possibility that there might be such a concentration of open jointing in that vicinity as would induce a definite intake and flow toward the drain pipes in the bed of the old drift.

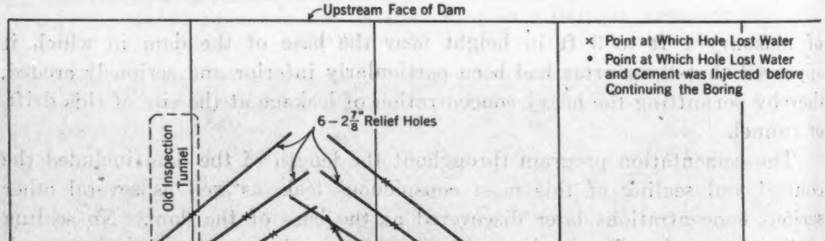

FIG. 13.—SHIRAWTA DAM, SHOWING METHOD OF STOPPING "BIG LEAK" AT STATION 6.5.

A supply of coal ashes mixed with top-soil and a small "sweetening" of cement was assembled on the parapet and sifted down into the water against the dam directly above the suspected area of intake. A dose of the fluorescein dye was also included on the chance of tracing the color in the gushing leaks. On first trial the dye showed color within a few minutes. This was followed after a time by a trace of color in the soil, and continuing the process for some hours produced a marked reduction of flow and pressure in the 6-in. drain pipes.

This important checking of the leakage along the path of the old tunnel-drift greatly facilitated the further progress which consisted of drilling a series of six borings, 3 in. in diameter, from the down-stream toe at about 3 ft below the tunnel floor and 25 ft each side of its axis. Entering the dam at 45°, these holes converged in two sets of three holes, one in each flank, to an intersection under the tunnel containing the 6-in. pipes at a distance of 25 to 30 ft from the down-stream face of dam (see Fig. 13).

The effect of these borings was first to take away most of the remaining flow from the 6-in. pipes. Then they were grouted (with a 1:1 mix) one at a time, with controlled interconnections while using the opposite set of borings for pressure relief as needed. To make the job complete a similar group of borings was made a little higher up for sealing the entire tunnel area. These converging holes, supplementing the numerous previous holes in the regular and special series, had the effect of stopping all sub-surface leakage and solidifying the entire base of the dam up to the floor of the tunnel.

Thereafter, it became a relatively simple, and personally comfortable, matter to seal the ground above the tunnel by pressure grouting through the top holes, with a regulated vent through the 6-in. pipes. Finally, all leakage through the drain pipes and the masonry joints of that vicinity was stopped in this manner (see Fig. 7).

Other Bold Leaks.—In attempting to grout primary top holes between Station 30.0 to 34.0 (see Figs. 14 and 15), it was found impossible to hold the cement in the dam merely by caulking down-stream face-joints above the ground surface, and the usual methods failed to stop the boils rising through the back-filling at the toe of the dam. The toe of the dam, at 15 ft below the back-filled surface, was explored by digging trenches as required. A number of bad leaks were exposed, partly in the lower face-joints; the worst was a leak that rose from a bed-rock fissure 3 ft beyond the toe (see Figs. 14 and 15). Another connected series of leaks was discovered directly on the surface of the bed-rock, under the toe of the dam which, for some distance, was found to rest on the flat smooth surface of rock without any indentation or key seating. These leaks in and on the rock could not be caulked, even against the residual of static reservoir pressure. They were controlled successfully by trapping the flow in a series of 2-in. and 3-in. pipes in the bottom of the trench under concrete placed to serve also as a toe retainer for the dam in the rock-bound trench.

The pipe traps, rising to the surface, became at first the pressure relief vents for the water, and then for grout injected through top holes in the dam. By throttling these relief pipes as the grout discharge thickened and finally

closing them, or extending a succession of stand-pipes until the pressure became balanced at 30 ft above ground, this section of the dam was made tight, and all toe leaks disappeared.

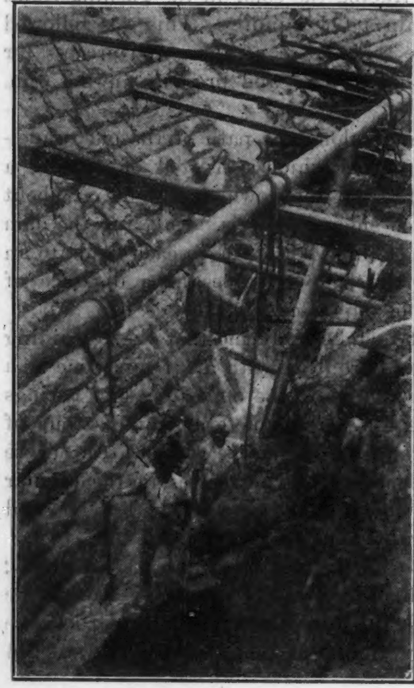

FIG. 14.—SHIRAWTA DAM SHOWING SECTION (STATIONS 30 TO 35) WHERE WORST GROUND AND TOE LEAKS OCCURRED.

FIG. 15.—SHIRAWTA DAM; ANOTHER VIEW OF STATIONS 30 TO 35.

EXPERIMENTAL INJECTIONS

The Interior Tube Method.—Several trials were made at Shirawta and at Walwhan with a small injection pipe extending nearly to the bottom of the boring and fitted with a gland or loose stuffing-box at the top, the object being to consolidate or seal the ground from the base up. This method has succeeded in foundations where raveling ground, soft rock, or sand has required a degree of solidification in relation to other features. In the dams treated in this paper, however, the trials demonstrated that the method was less effective than the adopted standard procedure with full boring under initial pressure—variable and adjustable but continuous, from the top.

Lubrication Methods.—After the primary and secondary injections of cement grout had left something to be desired in the way of more complete sealing of residual seepage, an effort was made to induce penetration of the finer voids by the addition of some form of lubricant to the cement grout. It was suggested that compressed air might have some such effect, but when

tried, produced no perceptible benefit. Calcium chloride solution mixed in the grout was no better—in fact, it made the worst record, presumably on account of its acceleration of the set. A soda-silicate, water-glass, solution appeared to help a little, but the average record of the eighty-seven borings under trial indicated that the straight 4% cement-water grout under the usual direct pumping pressure gave slightly better results than were secured with any attempted lubrication.

Chemical Treatment

Following the trials of "lubrication," as a last resort, and for the information to be derived, an attempt was made to substitute for the cement a combination of commercial salts which were reputed to have been highly beneficial in high-pressure injections for sealing wet ground in mines and shafts. In this case a section of Shirawta Dam was selected, 300 ft long, which had first been fully treated in five series of borings with the regular cement injections, and yet continued to show slight seepage over much of the down-stream face.

New borings were drilled from the top to bed-rock, at 12½-ft intervals, in this section of the dam; there were twenty-five holes in all, as exhibited in Table 1 (Items Nos. 23, 24, and 25). These borings were injected first with a soda-silicate solution immediately followed by an alumina sulfate solution. The silicate was supposed to smooth the path for the sulfate, which crystallizes and solidifies in place. Injections were made first in stages of 15-ft depth from the top down; then in 50-ft stages without any perceptible difference in effect. The maximum pressure was 80 lb per sq in. The result was a visible improvement in sealing the residual seepage, but not practically measurable in the small flows involved, which were complicated by the time interval and reservoir levels.

The conclusions regarding the chemical treatment were that:

(1) It is only applicable after thorough cementation in the usual manner for stopping all large leaks; hence, a complete re-boring of the dam is necessary, with spacing not greater than 12½ ft;

(2) The improvement in density and strength of the masonry is inappreciable;

(3) The reduction of seepage below that already accomplished by cementation is not appreciable, its durability is not proved, and the effect of the injected salts on the surkhi mortar and cement grout is a matter of doubt; and,

(4) The small immediate or visible improvement will not justify the expense.

It is probable that the reputed success of "chemical treatment" has been in rigidly confined ground where pressures of several hundred pounds per square inch were permissible for driving the fluid, with no granular solids in suspension, into the smallest passages and interstices. Manifestly, such high pumping pressure could not be permitted in a rather loosely jointed structure of relatively small dimensions resisting unbalanced static pressures, such as these dams.

The suggested limitation of benefits by the cementation process alone, opens a further field of usefulness for cement chemists and testing laboratories. A safe and reliable means of sealing the voids in otherwise completed masonry structures is needed, in order to make them virtually impermeable by water. This problem involves the production, at moderate cost, of a substance of demonstrable durability, having the property of setting or hardening in place without loss of volume or injurious reaction to any masonry constituent, and which can be carried in solution in water under pressure while maintaining a fluidity equal to, or greater than, water itself.

It may be claimed by some that the current patented or proprietary waterproofing compounds would serve for this purpose; or, it might be imagined that the old Sylvester wash of soap and alum could be injected under pressure where any solids in a water vehicle would not go; but the field is open for modern scientific experimenting.

Cementation of the Other Dams

Walwhan Dam.—The foregoing has been largely in relation to Shirawta Dam where most of the problems were first encountered. At Walwhan Dam (see Figs. 16 and 17) the same equipment and methods were used, with about the same results, as shown in Tables 1 and 2. The thin section, arched design, low specific gravity of rock and masonry, and the more general spread rather than concentration of leakage at Walwhan, introduced special difficulties requiring modification of methods.

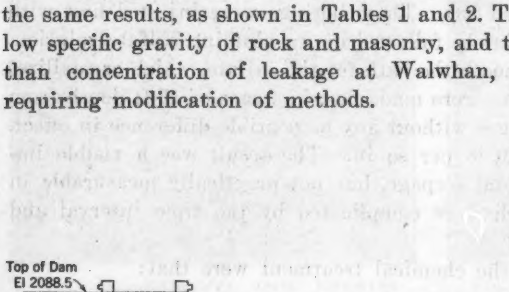

FIG. 16.—TYPICAL SECTION, WALWHAN DAM, SHOWING POSITION OF THE SEVERAL SERIES OF BORINGS.

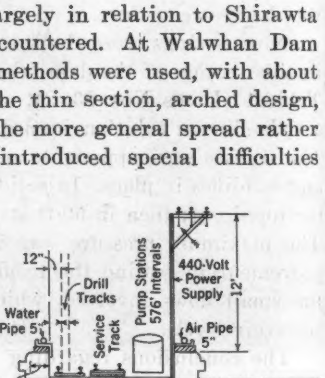

FIG. 17.—TYPICAL SECTION, WALWHAN DAM, SHOWING ARRANGEMENT OF PLANT.

The principal difficulty was to place the borings in positions where they would be sufficiently covered by masonry to hold the grout under any effective pressure (see Fig. 18). The top 20 ft of dam had to be treated as a first stage under low injection pressure, and even then this upper stage gave insufficient

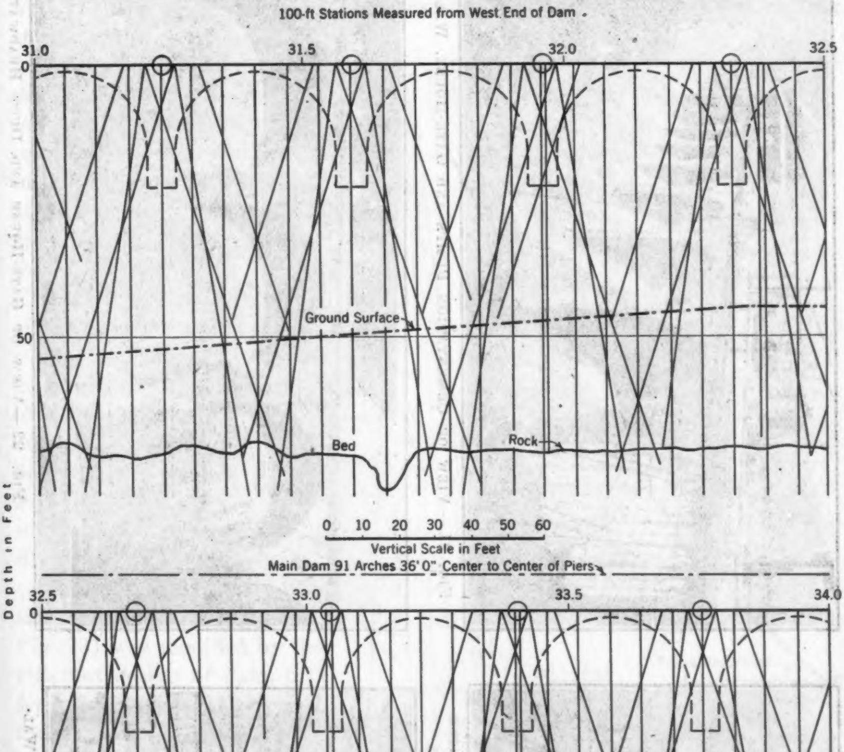

FIG. 18.—TYPICAL SPACING OF TOP BORINGS, WALWHAN DAM.

support under the crown of the arches; hence, the boring was required to be done only with the shot drills, in order to avoid vibrations, and at such slopes as would pass through the piers and spandrels to reach the lower measures of the masonry where higher injection pressure was needed. Views of this work are presented in Figs. 19, 20, 21, and 22. Cement and grout are plainly visible in Figs. 20 and 21.

FIG. 19.—DRILLING Z-HOLES, WALWHAN DAM; PERCUSSION DRILLS ON JUMBO STAGE.

FIG. 20.—GROUTING LOWER Z-HOLES IN PAIRS, WALWHAN DAM.

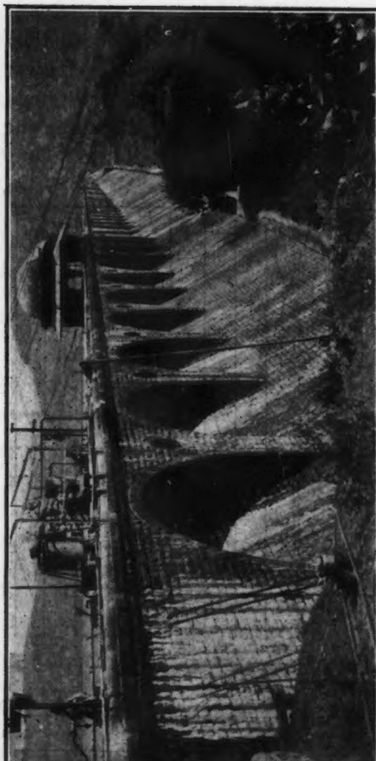

FIG. 21.—VIEW OF CEMENTATION PUMPS AND GATE-HOUSE, WALWHAN DAM.

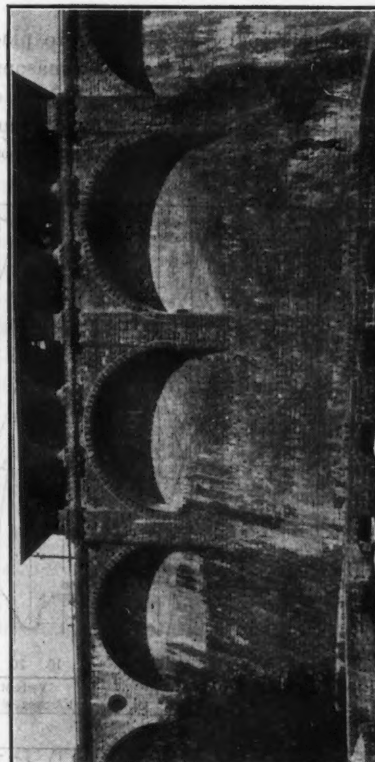

FIG. 22.—VIEW OF GATE-HOUSE AND DUCT HEADWAY.

Considerable joint caulking was required in down-stream face-joints, but this was done more readily with strips of sacking and less forcible wedging than was required at Shirawta. Grout was prevented, fairly well, from escaping to the lake by the reservoir head against a thorough pointing of joints which had been done several years previous—an advantage which was lacking at Shirawta.

The travel of grout through the wall, with interconnection of borings and venting of water and grout in the face-joints, 50 to 75 ft distant from borings under pressure, was more general than in the other dams. The progressive tightening of the masonry, with diminishing consumption of cement, in the successive series of borings and injections was equally significant; and, finally, the leakage index, 90% reduction, was substantially the same as at Shirawta.

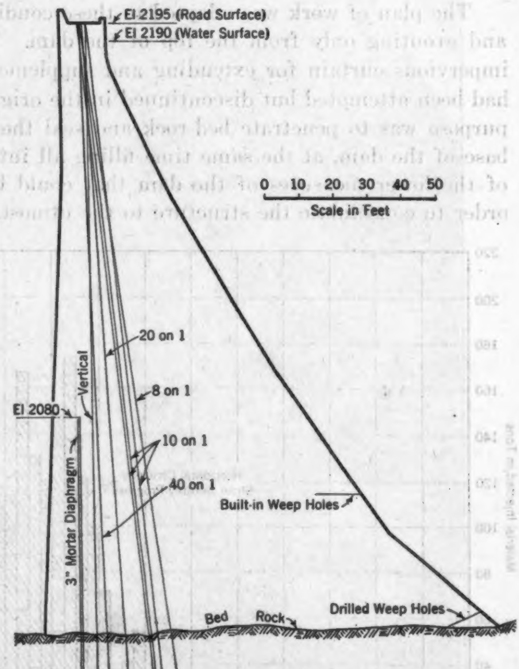

FIG. 23.—TYPICAL SECTION (MIDDLE) OF THOKERWADI DAM, SHOWING POSITION OF THE SEVERAL SERIES OF BORINGS.

Thokerwadi Dam.—The situation at Thokerwadi (see Fig. 23) was modified by the different design of dam, different types of masonry (all surkhi mortar, however), harder and heavier rock in the aggregates and in the foundation, together with the great difference between the high middle section and the low end sections, as separated by the buttresses which had been built over the shrinkage cracks in the dam as originally constructed. The record of cement consumed is shown in Fig. 24.

Peculiarities of the design (gravity type) included a 3-in. mortar diaphragm plastered against the hearting masonry aproximately 10 ft in from the up-stream face and only from bed-rock up to a level 115 ft below the top of the dam. The design also contemplated draining the masonry through built-in ducts emerging in 4 by 8-in. rectangular weep-holes through the down-stream face. Fig. 25 shows one line of these weep-holes which carried a large part of the total leakage through the dam (4.0 cu ft per sec, which was reduced to 0.25 cu ft per sec by the cementation process). Leakage through these weep-holes was entirely stopped by the interior injections of cement (see Fig. 26).

Another large part of leakage emerged in the re-entrant angles of the buttresses where similar openings had been made for venting any water reaching the buttress through the shrinkage crack. The upper part of the dam was built of surkhi concrete with the heavy basaltic aggregates, including crushed sand. This part was first bored and grouted, with consumption of but little cement.

The plan of work was adapted to these conditions, which indicated drilling and grouting only from the top of the dam. The objective was to form an impervious curtain for extending and supplementing the "diaphragm," which had been attempted but discontinued in the original construction. The further purpose was to penetrate bed-rock and seal the bedding joint throughout the base of the dam, at the same time filling all interstices in the rubble masonry of the lower measures of the dam that could be reached by such borings in order to consolidate the structure to the utmost. (See Figs. 23 and 26).

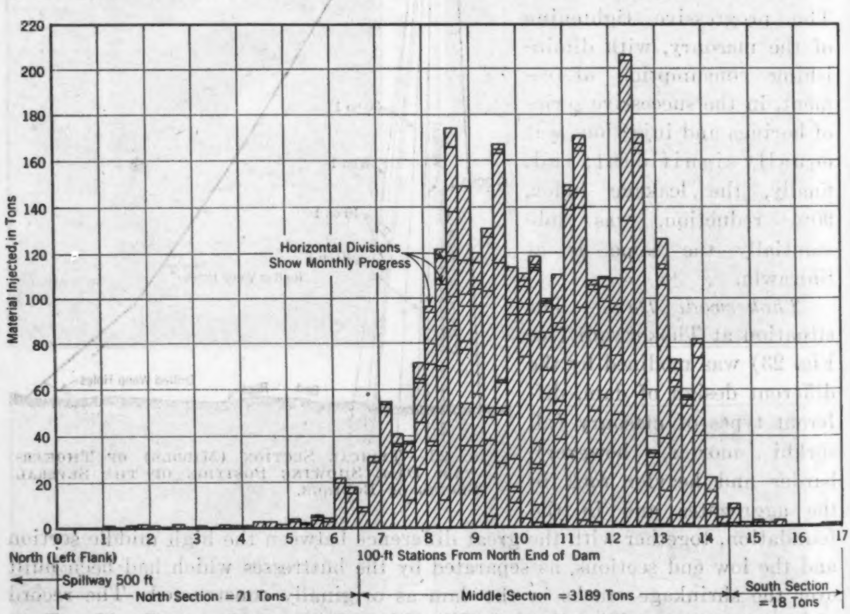

FIG. 24.—CEMENT CONSUMED, THOKERWADI DAM, IN LONG TONS (2 240 LB) PER 25-FOOT LENGTH OF DAM.

More than one-half the length of the dam (both ends) was of concrete, well seated in bed-rock, and the borings at 5-ft spacing consumed relatively little cement. The shrinkage cracks through the dam at the two buttresses, 700 ft apart, were grouted and well sealed by injection through borings intersecting each crack at 50 ft, 100 ft, and full depth. The section between the buttresses, 150 to 190 ft high, was of the same concrete in the top 100 ft; below that level the masonry was of loose, jointed rubble that raveled in the borings, permitted loss of drilling water, and required injection in several stages of depth.

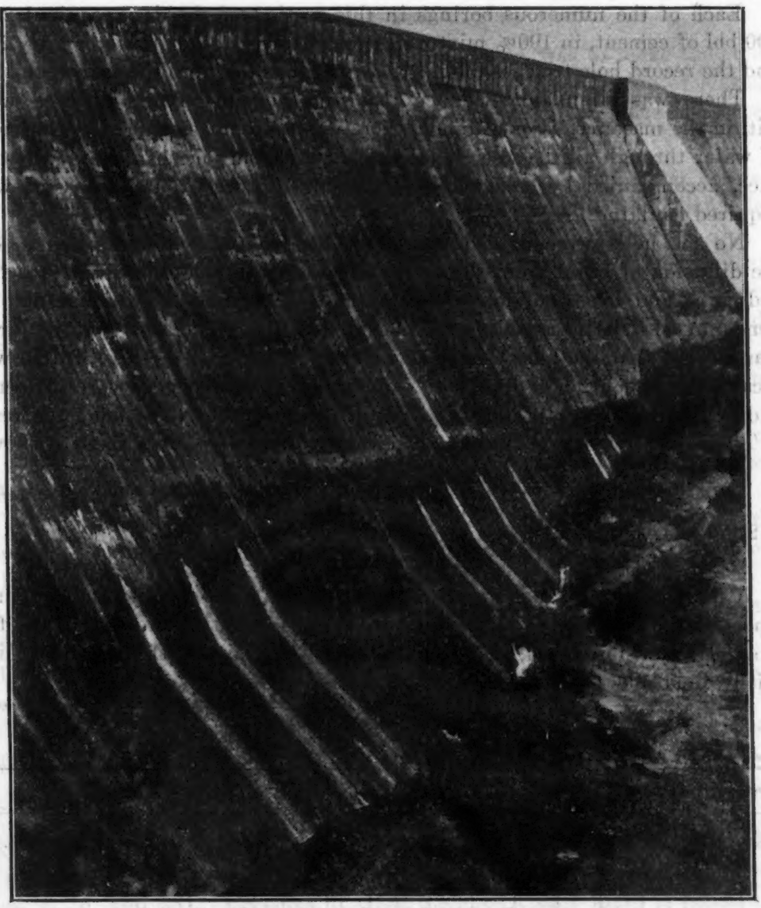

FIG. 25.—THOKERWADI DAM, BEFORE CEMENTATION.

FIG. 26.—THOKERWADI DAM, AFTER CEMENTATION.

Each of the numerous borings in this section of the dam required 200 to 300 bbl of cement, in 100% mixture (1 part cement to 1 part water by weight), and the record hole took 700 bbl. No cement was lost into the lake at this dam.

There was abundant evidence that the cement became diffused widely within the masonry. For example, there was a progressive decrease in leakage of water through all the weep-holes and through the joints in the down-stream face, accompanied by a showing of cement in these vents, many of which required caulking.

No toe holes were drilled for injections from the down-stream face, as the diffusion of grout from the several series of top holes appeared to seal the bed joint and to consolidate the masonry satisfactorily. A remarkable evidence of the latter result was the progressive tightening of the drilled masonry, so that the holes ceased to ravel and cave, and the loss of drill water became infrequent. The final series of borings produced good cores of cement and refused thin grout under maximum pressure. On the whole, the results at Thokerwadi were considered rational and satisfactory, with the best showing of the three dams in $\frac{\text{cement}}{\text{masonry}}$ ratio and reduction of leakage (Tables 1, 2, and 3).

Record and Costs

A detailed record and report of all the work included a drilling log, and a report of the behavior of all borings during the injection process, in the form of a "cementation record," as shown in Fig. 27. The cost of the work is listed briefly in Table 1 (Items Nos. 27 to 34).

(Form No. [illegible]

WALWHAN DAM. SHEET No.

C. P.—200—9-33.)

RECORD OF CEMENTATION.

HOLE NO.	DEPTH FT.	HRS. WORK.	DATE DONE.	C/W RATIO (WEIGHT).		CEMENT-TONS.			PRES.	WATER LOSS G.P.M.	LEGEND.
				BEGIN.	END.	INJ.	LOST.	TOTAL.	LBS.		
SHEET TOTAL											
TOTAL											

FIG. 27.—SAMPLE SHEET FOR DAILY RECORD USED AT ALL DAMS.

These data of over-all costs cover the total expense of the work under the five items of Plant, Supplies, Materials, Labor, and Supervision. These cost figures would be applicable for estimation purposes in other parts of the world, by the use of a suitable factor of performance in drilling, under any local conditions. Knowing the cutting speed of the types of drill to be used in the local rock and masonry, either by previous experience or by a trial run, consideration of Indian labor cost would be eliminated.

The cost of plant, equipment, stores, and repairs, as shown in Table 2, should represent a maximum expense for these items elsewhere. The cost of cement in India is about the same as the average in the United States. The cost of supervision, as shown in Table 2, is a reasonable value for use anywhere to cover adequate direction and inspection of this special type of operations.

Conclusions

The main objective of this work, on all three dams, was to make the structure as solid and stable as practicable. Incidentally, the reduction of leakage, or seepage, was an object, index, and measure of success. Evidences of success in the main objective may be reviewed, as follows:

(*a*) The marked change in the character of drilling after the primary grouting, as demonstrated by increased firmness of masonry with much less raveling and choking of the borings;

(*b*) The much less frequent loss of drilling water in the secondary and subsequent series of borings as cementation progressed, until in the final series most holes were water-tight, or consumed a minimum of cement under maximum pressure;

(*c*) The greatly decreased number of cases of interconnection of borings and of leaks to the lake, or otherwise, beyond easy control;

(*d*) The recovery of numerous cores showing good penetration of cement into fissures and interstices. (The cement in these cores was found to be well hardened in place, promising durability and permanence);

(*e*) The progressive decrease in cement consumed by the successive series of borings, particularly from primary to secondary holes, shown in Table 3, until at last many holes refused grout immediately; and, finally,

(*f*) The reduction of leakage by more than 90% is good evidence of success in solidification.

The weight of evidence seems to warrant the conclusion that these dams have been stabilized and secured (largely by isolating and restricting the continuity of uplift areas), a gratifying conservation of water has been effected, and the total cost of the work was reasonable and well justified.

Reflection on the Stability of Gravity Dams

It will be observed that these dams of the "gravity" type, straight in plan, are of more slender profile than could be approved under orthodox analysis. (Figs. 1, 16, and 23). Probably it could be shown that there is tension in the up-stream masonry, which is certainly not of a character to withstand appreciable tensile stress.

Under the best conditions of workmanship with the given materials it could scarcely be assumed that any section of these dams would be of the monolithic character contemplated in the ordinary computations for overturning moment; and, yet, all these dams have been in successful service about fifteen years, with such deterioration from their original conditions as is evidenced by the increasing percolation during that period. Against this background, the observations incidental to the cementation process have tended to overturn, undermine, or invalidate the observer's conceptions of why a dam stands up. His net reaction is in accord with the declaration of "the necessity for abandoning the middle-third theory and the sliding factor as useful elements in masonry dam design."[2]

[2] "Stability of Straight Concrete Gravity Dams," by D. C. Henny, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1041.

In view of all the phenomena presented by the cementation of these dams, however, the writer hesitates to follow Mr. Henny all the way regarding the critical question of "uplift." Rather, it might be concluded from the evidence of these thousands of borings, under their manifold pressure testing, that the occurrence, persistence, and practical effect of uplift have each been unduly magnified in the fundamental assumptions of dam design. On the other hand there would appear, from these experiences, to be good reasons for the adoption of a rational shear factor in substitution or modification of the usual sliding factor. Moreover, the common assumption of a monolithic cantilever section comes under suspicion.

While the writer was stationed for some weeks at the toe of Shirawta Dam, below a full reservoir, with the monsoon pouring water on his head and the "big leaks" sloshing water on his feet, watching the effect of pumping water and cement into the middle of the structure, he could perceive no good reason why that dam should remain stable; but it did remain stable, although sections of it 25 to 50 ft long appeared to be merely floating in water and grout. It seemed as if it had uplift enough to move it.

For the most part the bedding on rock was good and the vertical bonding of the rubble masonry was fair; but there must have been planes where uplift operated to reduce the sliding resistance. The saving factors must have been shear resistance and horizontal beam, or voussoir resistance arching between abutments of contiguous sections of the dam which had a little surplus of stability—possibly by taking the resultant thrust longitudinally.

At long last, the one conspicuous result of the cementation program was to localize, delimit, or destroy the continuity of all considerable areas of hydrostatic uplift; but, of course, this result should have been accomplished in the original construction. The lesson to be derived from this experience is, that builders of gravity dams may take some chances on their design of profiles and the governing theories of stability, but they should not take chances on the soundness of their materials or the competence of their workmanship and field direction.

Designers should "keep their feet on the ground" and beware of using values of uplift from bed-rock, based on over-inflated formulas. They should also keep their eyes critically on field practices, to the end that the essentials of durable construction shall become incorporated with approved mechanical principles.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

WEIGHTS OF METAL IN STEEL TRUSSES

By J. A. L. Waddell,[1] M. Am. Soc. C. E.

Synopsis

From the ten sets of curves in this paper a designer can read the ratio of the weight of metal in a truss (per linear foot) to the total vertical load (live, impact, and dead loads per linear foot) for which the truss is to be proportioned. They apply to simple truss, cantilever, and arch spans, for railways, highways, and combinations of both, designed either with carbon steel or with silicon steel.

Modifying formulas or data are given to cover uneconomic proportions, the use of high-alloy steels, and variations in the specified intensities of working tensile stress for carbon-steel structures. There are also four sets of curves showing total weights of metal per linear foot for various classes of bridges. These curves are inserted for the purpose of enabling the computer to determine his trial total load.

The paper concludes with a table indicating the results of the "spot-checking" of the percentage ratios, shown on the diagrams, by means of a score of spans for which the truss weights had previously been accurately determined. Only the last two of these cases were utilized in the preparation of the ratio curves; hence the others serve as an absolutely unbiased check on the accuracy of the diagrams.

Introduction

If a bridge computer were able (generally within a minute or so) to find by diagram, for spans of any reasonable length and for the usual types of structure, the ratio of weight of metal per linear foot in a truss to the total vertical load per linear foot for which that truss is to be designed, he would be saved much time and trouble. The writer has attempted to supply that need by collating extensive office records accumulated in practice, and plotting them in the form of curves for the use of the designer.

Note.—Discussion on this paper will be closed in May, 1935, *Proceedings*.

[1] Cons. Engr., New York, N. Y.

The results obtained are sufficiently accurate for preparing preliminary estimates of cost and for determining dead loads. In fact, they are so accurate that, when properly modified for variations in the intensities of working tensile stress due to using design specifications other than those recommended by the writer,[2] it is probable that no recomputation will ever be necessary because of any serious discrepancy between the assumed and the calculated dead loads. It would be futile to attempt to attain greater accuracy, because each bridge of any importance has some special individuality of its own, tending to cause slight variations from the most accurate of any curves of weight-of-metal ratios that could be made. Furthermore, each designer has personal idiosyncracies that affect the weights of the trusses he computes, and there is quite a perceptible difference in the metal weights between structures which are truly first-class in every particular and those of only mediocre excellence, or those that have been "trimmed" to the limit.

Extent and Limits of This Investigation

It will be noted that Figs. 1 to 5 include simple truss spans, cantilevers of both Type *A* and Type *C* (Fig. 6), and arches; but they embrace neither suspension bridges nor bascule spans. A little thought will convince any one with experience in bridge designing that it would not be feasible to plot the last two types.

The curves cover trusses of both carbon steel and silicon steel; and formulas are given herein for converting values of the weights of metal for silicon steel to the corresponding weights for all feasible alloy steels, up to those having an elastic limit of 100 000 lb per sq in. They embody single-track and double-track railway bridges, modern highway bridges of 20-ft clear roadway without sidewalks (the cheapest legitimate type of structure), and those of 40-ft clear roadway with two 5-ft sidewalks (the most common dimensions for first-class structures). For any other cross-section of floor, the desired "percentage ratio" can be found by interpolation or extrapolation, remembering that, in establishing the equivalent total width of floor, the sum of the widths of the sidewalks must be divided by two.

In the preparation of Figs. 1 to 5, the live loads for railroad bridges were those identified as Class 60 in "Bridge Engineering,"[3] and those for highway bridges were taken from curves presented elsewhere by the writer.[4] It will be noted that only three standard live loads have been utilized.

In Figs. 1(*e*), 1(*f*), 3(*a*), and 5, the truss weights include the weight of the metal in the bents above the arch rings. The reason for inserting Figs. 3(*a*), 3(*b*), 4, and 5 is to aid computers in finding the trial total load per linear foot for any proposed structure; after doing which the "percentage ratio" can be ascertained from one of the first ten diagrams and another trial made, if necessary.

[2] "Bridge Engineering," by J. A. L. Waddell, M. Am. Soc. C. E., Vol. 1, Chapters XIV and LXVIII, John Wiley & Son, New York, N. Y., 1916.

[3] *Loc. cit.*, p. 103.

[4] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 823 (see Figs. 1, 2, and 3).

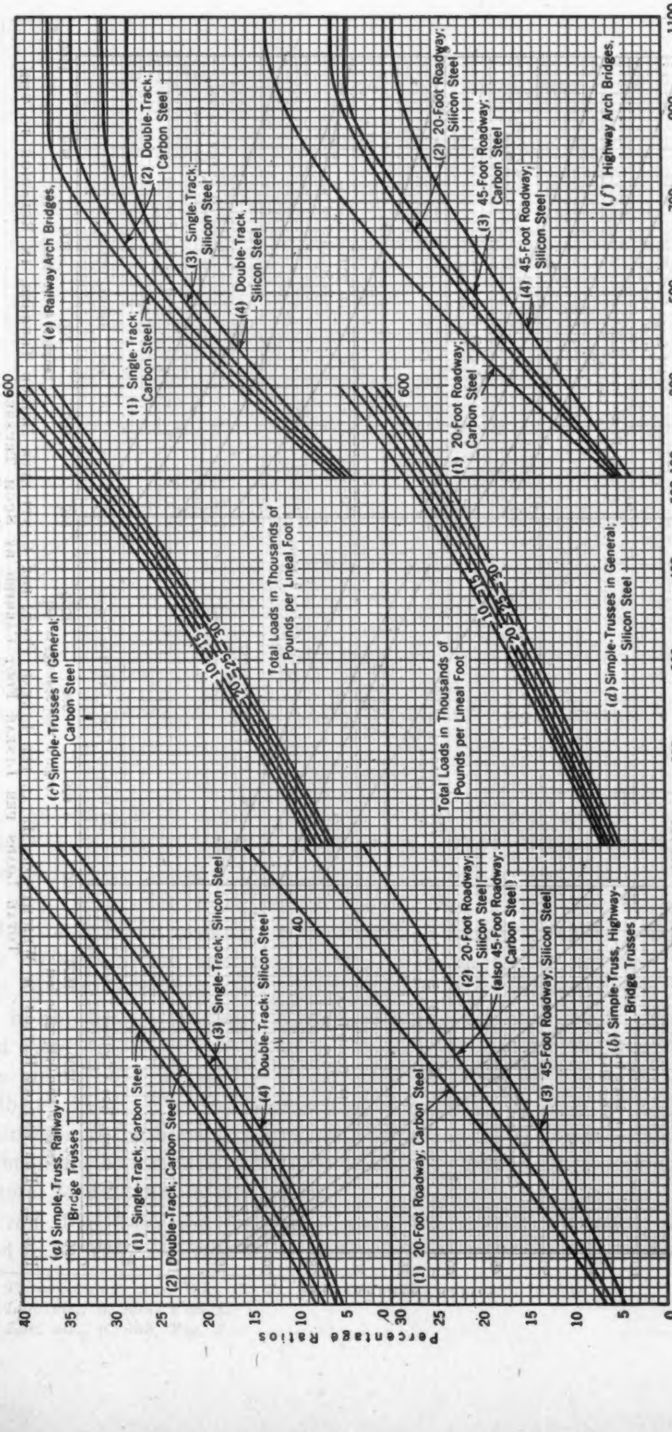

FIG. 1.—PERCENTAGE RATIOS OF WEIGHTS OF METAL PER LINEAR FOOT, FOR BRIDGE TRUSSES, IN RELATION TO THE TOTAL LOADS PER LINEAR FOOT CARRIED BY SUCH TRUSSES.

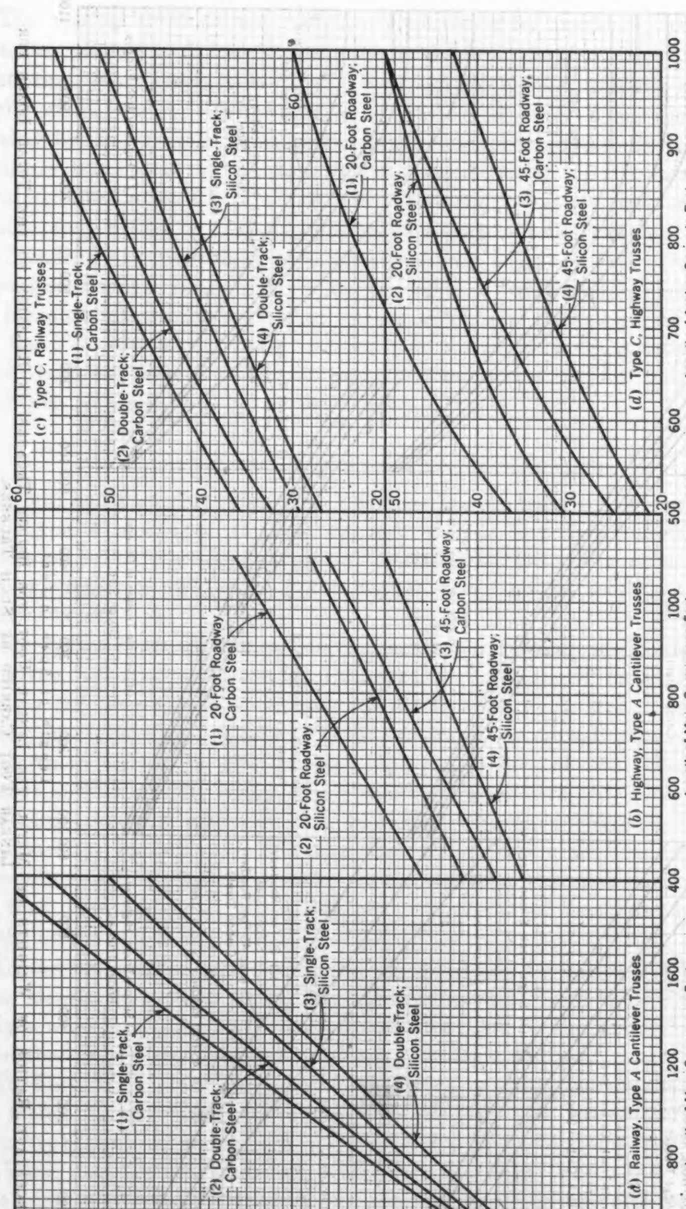

FIG. 2.—PERCENTAGE RATIOS OF WEIGHTS OF METAL PER LINEAR FOOT, FOR CANTILEVER BRIDGE TRUSSES, IN RELATION TO THE TOTAL LOADS PER LINEAR FOOT CARRIED BY SUCH TRUSSES.

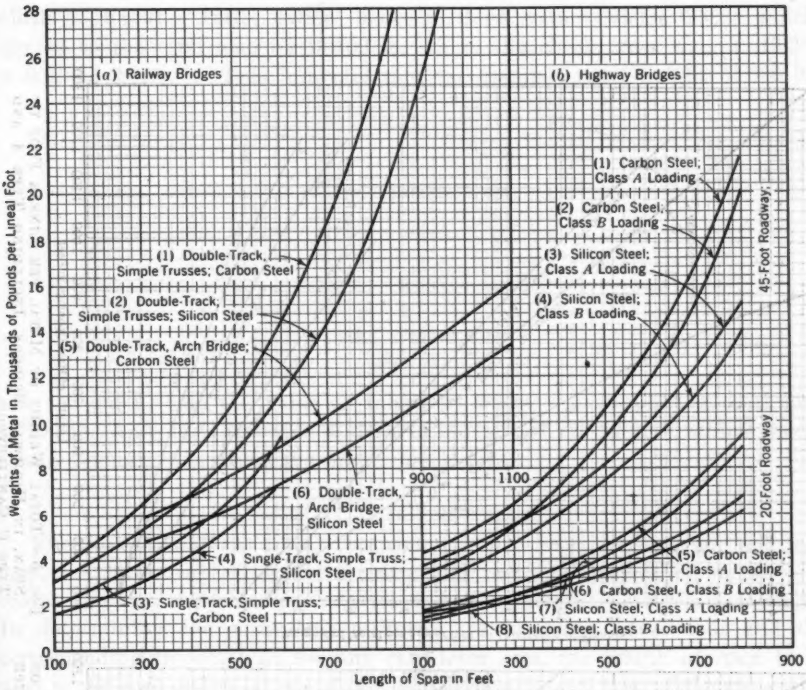

FIG. 3.—TOTAL WEIGHTS OF METAL PER LINEAR FOOT OF SPAN, IN ARCHES AND SIMPLE TRUSS SPANS.

EXAMPLE

As an example of how a computer would proceed with a new problem, consider the design of a 450-ft, simple-truss, highway span, with Class *A* loading, a 30-ft clear roadway, two 8-ft sidewalks, and trusses of silicon steel. The first step is to determine the character and weight per linear foot of the flooring for both roadway and sidewalks. Assume that this weight is found to be 4 300 lb. The next step is to determine the approximate weight of metal (generally carbon steel) in the floor system and hand-rails, as well as that in the lateral system, which should be of silicon steel. From a prepared table[5] the weight of metal in the floor system and hand-rails is found to be about 1 650 lb per lin ft, and that of the lateral system to be about 540 lb per lin ft. The live load would be found by proportion[6] to be 3 360 lb per lin ft, and the impact,[7] 13.3% of 3 360 or, say, 450 lb per lin ft. The truss weight might be assumed at 3 000 lb per lin ft. Adding these quantities makes the total load 13 300 lb per lin ft.

From Fig. 1(*b*) the percentage ratio for a 45-ft equivalent roadway is found to be 0.237 and that for a 20-ft roadway, 0.285; the equivalent roadway

[5] *Transactions,* Am. Soc. C. E., Vol. 98 (1933), p. 896, Table 2.
[6] *Loc. cit.*, p. 893, Fig. 1.
[7] *Loc. cit.*, p. 895, Fig. 3.

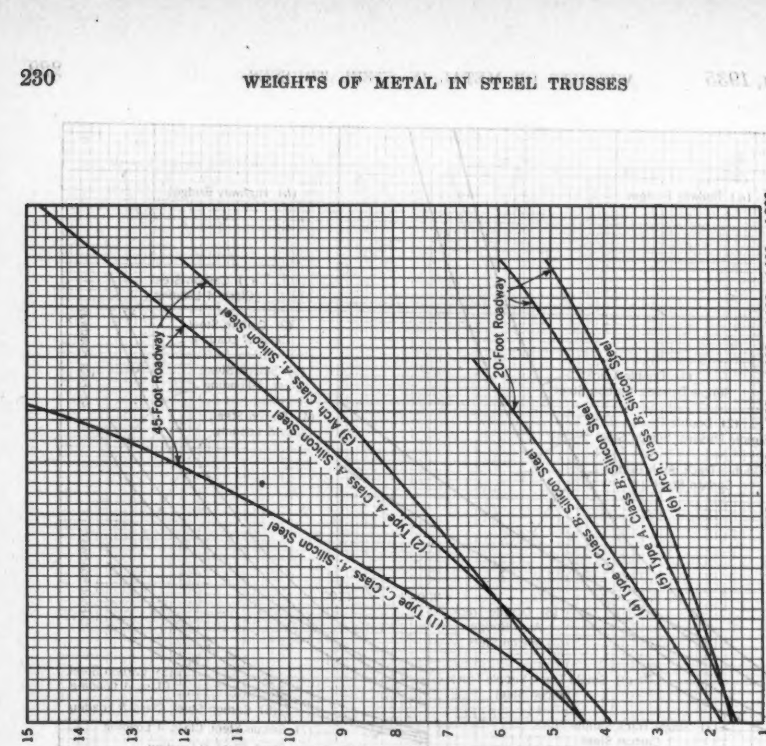

FIG. 5.—TOTAL WEIGHT OF METAL PER LINEAR FOOT OF SPAN, IN HIGHWAY BRIDGES, INCLUDING TYPE *A* AND TYPE *C* CANTILEVERS.

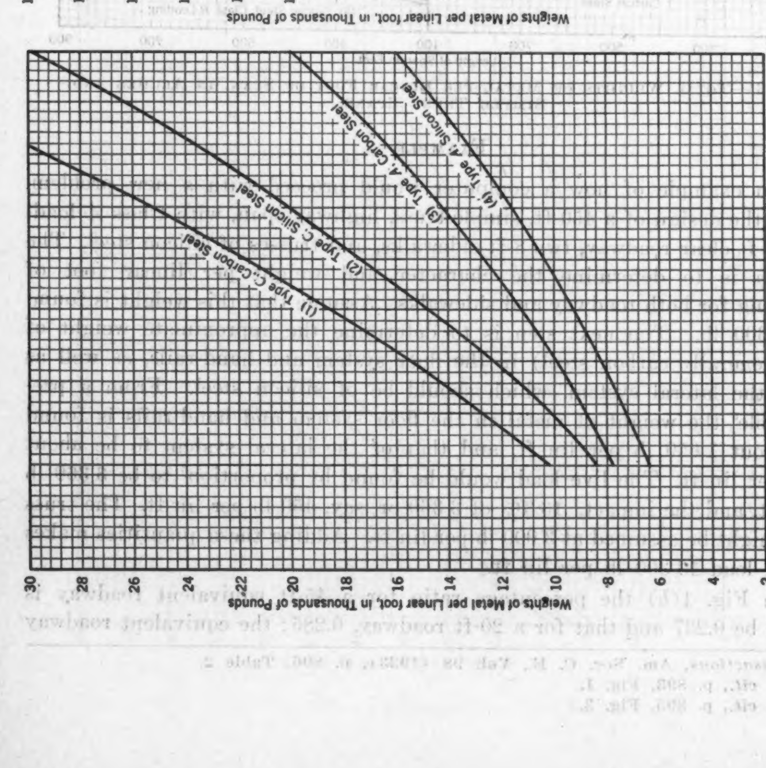

FIG. 4.—TOTAL WEIGHTS OF METAL PER LINEAR FOOT OF SPAN, IN DOUBLE-TRACK, RAILWAY, TYPE *A* AND TYPE *C* CANTILEVER BRIDGES.

width is 38 ft. Interpolation between these values yields 25%, which, applied to the total load of 13 300 lb, makes the check truss weight, 13 300 $\times$ 0.25 = 3 325 lb. This indicates that the assumed truss weight (3 000 lb)

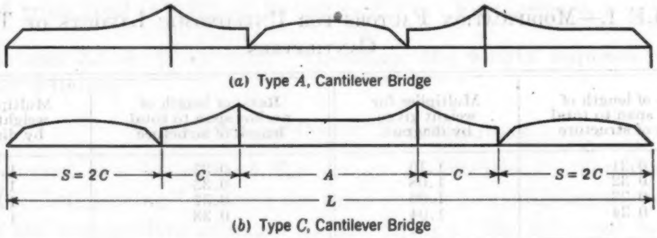

FIG. 6.

was too small, and the value is raised to 3 400 lb, making the total load 13 700 lb. As before, this would yield 13 700 $\times$ 0.25 = 3 425 lb, which checks closely enough to serve as the final truss weight.

TOTAL LOADS

The "total loads" per linear foot comprise live load, impact load, and dead load, the latter including the flooring and the total weight of metal. Of course, the truss dead load excludes the metal on the piers or in the anchorages. In determining the equivalent uniform live load per linear foot and the corresponding impact load for any cantilever arm, the length of span to be used is the sum of the length of one cantilever arm and that of the suspended span. In finding the average uniform live load plus impact, per linear foot, for the entire structure, it is not correct to take the direct average for the three distinct parts of it; each value computed must be multiplied by the total length that it governs, the three results added, and the sum divided by the total length of structure. The same method is to apply to the finding of the average total truss load per linear foot, and of the average weight of truss metal per linear foot, for any cantilever bridge.

MODIFICATIONS OF WEIGHT-CURVE FINDINGS

In determining the weights of metal for the various curves, economic proportions of layout were assumed. If uneconomic designs must be utilized, the weights found from these curves will need modification, as shown in the text following.

Type C Cantilevers.—For greatest economy of metal, the length of the anchor span should be 0.36 times the total length of the structure. If the proportion differs from this, the average weight of truss metal per linear foot will have to be multiplied by one of the quantities given in Table 1.

Arch Bridges.—For the greatest economy of metal in two-hinged and three-hinged arch bridges the ratios of rise to span are as follows:

Solid-rib structures ....0.175 to 0.225
Braced-rib structures ....0.200 to 0.250
Spandrel-braced structures, with hinge above. .0.225 to 0.275

For hingeless arches each of the foregoing values must be increased by 0.05. The variations in weights of metal in arches caused by the adoption of uneconomic ratios of rise to span length have never yet been determined.

TABLE 1.—MODIFICATION FACTORS FOR UNECONOMIC LAYOUTS OF TYPE C CANTILEVERS

Ratio of length of anchor span to total length of structure	Multiplier for weight given by diagram	Ratio of length of anchor span to total length of structure	Multiplier for weight given by diagram
0.31	1.10	0.35	1.01
0.32	1.08	0.36	1.00
0.33	1.06	0.37	1.05
0.34	1.04	0.38	1.19

For this investigation it is not worth while to make special designs for a number of arch bridges in order to find them—especially in view of the fact that the economic ratio for any case has a fairly wide range (0.05), which is about 11% either above or below the average of the economic ratios.

Uneconomic Truss Depths.—The variations in weight of metal in simple trusses caused by the adoption of uneconomic truss depths almost invariably occur in deck spans, due to a restricted vertical distance between clearance line and grade; and, in these cases, parallel chords are always used.

The correcting ratio, r, is determined thus: Let w = weight of metal, in pounds per linear foot, for a truss of economic depth, d; and w' = weight for any other depth, d', less than d.

The weight, w, is about equally divided between the chords and the web; the chord weight varies almost inversely as the depth; and the web weight varies almost directly with the depth; hence,

$$w' = \frac{w}{2} \times \frac{d}{d'} + \frac{w}{2} \times \frac{d'}{d} \quad \text{.....................(1)}$$

and,

$$r = \frac{w'}{w} = \frac{1}{2}\left(\frac{d}{d'} + \frac{d'}{d}\right) \quad \text{....................(2)}$$

WEIGHTS FOR ALLOY-STEEL TRUSSES

To find the weight of metal for any alloy-steel truss, with an elastic limit, s_y, all that is necessary is to ascertain from Figs. 1 to 5 the weight of truss metal for silicon steel, and multiply it by:

$0.3 + 0.7r$, for medium spans (less than 500 ft);
$0.25 + 0.75r$, for fairly long spans (500 ft to 1 000 ft); and
$0.2 + 0.8r$, for very long spans (more than 1 000 ft).

In the foregoing values, $r = \dfrac{45\,000}{s_y}$ (45 000 being the elastic limit for silicon steel). For example: What is the linear weight of truss metal in a simple-truss, double-track railway span of 500 ft, when an alloy steel of

70 000 lb elastic limit is utilized, the total load per linear foot being 25 000 lb for a silicon-steel bridge?

For a silicon-steel bridge, Fig. 1(*a*) yields a ratio of 0.272; therefore, the weight of truss steel is $0.272 \times 25\,000 = 6\,800$. Furthermore, $r = \dfrac{45\,000}{70\,000} = 0.643$; and $0.3 + 7r = 0.75$. Therefore, the weight required $= 6\,800 \times 0.75 = 5\,100$ lb.

Adaptation to Other Specifications

The curves can be utilized for design specifications other than those of "Bridge Engineering," because the main factor in modifying the weight of metal is the comparative averages of the ratios of the principal intensities of working stresses in the two cases. The final average is almost exactly that of the two specified intensities of working tensile stress. The intensity specified in "Bridge Engineering" for carbon steel is 16 000 lb per sq in., and that for any more modern set of bridge specifications is likely to be greater—in most cases as much as 18 000 lb per sq in., and in extreme cases, 20 000 lb per sq in. If the new intensity is denoted by s_t, then $\dfrac{16\,000}{s_t}$ will be the value of the ratio, r.

If Figs. 1 to 5 indicate a weight, w, for the writer's specifications, the corresponding weight for the other specifications will be:

$w' = w\,(0.3 + 0.7r)$ for short spans (less than 500 ft);
$w' = w\,(0.25 + 0.75r)$ for medium spans (500 ft to 1 000 ft); and,
$w' = w\,(0.2 + 0.8r)$ for long spans (more than 1 000 ft).

For instance, if the intensity of tensile stress for carbon steel in the other specifications is 18 000 lb per sq in., for a span of medium length, r will be $\dfrac{16\,000}{18\,000} = 0.889$; and $w' = w\,(0.25 + 0.75 \times 0.889) = 0.917\,w$.

In addition to a double checking by the writer of all the calculations made for this paper, the curves have been "spot-checked" by his firm's office force through reference to recorded results of both actual and computed weights of metal, for structures engineered during the last two decades by both himself and his present firm; and the outcome of this spot-checking has been eminently satisfactory, as can be seen by reference to Table 2. As the last two cases were the only ones of this table used in preparing the first ten of the diagrams given herein, the other seventeen cases provide an unbiased check upon their reliability.

In the Portsmouth (N. H.) lift-span, for the sake of æsthetics, the chords were made parallel, whereas the top chords of the two flanking spans, of like length, are polygonal. The lack of economy in the parallel chords (see Column (11), Table 2) accounts for the pronounced variation between the computed ratio and the one found from the diagram. The Jacksonville (Fla.), the Tombigbee River, and the Cape Cod Canal Bridges were very carefully computed; but they have not yet (1935) been built.

TABLE 2.—COMPUTATIONS TO CHECK THE ACCURACY OF FIGS. 1 TO 5

Structure	Type of bridge	Type of span	Width of roadway, in feet*	Foot-walks: Number	Foot-walks: Width, in feet	Span length, in feet	Total load, in pounds per linear foot	Truss Metal: Weight, in pounds per linear foot	Truss Metal: Kind of metal	Percentage Ratios: Computed	Percentage Ratios: From the curves
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Springfield, Mass.	Highway	Fixed	54	2	8	163.3	5 730	858	Carbon	15.0‡	12.9
Saratoga Lake, N. Y.	Highway	Fixed	20	1	5	200.0	6 500†	800†	Carbon	12.4†	12.8
Tombigbee River, Ala.	Railway	Fixed	One*	..	..	207.0	14 100	2 160	Carbon	15.3	14.5
Tombigbee Piver, Ala.	Railway	Lift	One*	..	..	161.0	15 940	1 810	Carbon	11.4	11.3
Albany, N. Y.	Highway	Fixed	42	2	6	222.0	18 320	2 174	Carbon	11.9	12.0
Albany, N. Y.	Highway	Lift	42	2	6	341.0	19 400	3 090	Silicon	16.0	16.0
Jacksonville, Fla.	Highway	Fixed	40	2	6	264.0	14 280	2 400	Carbon	16.8	16.3
Jacksonville, Fla.	Highway	Fixed	40	2	6	264.0	13 680	1 800	Silicon	13.2	12.5
Portsmouth, N. H.	Highway	Fixed	28	1	6	297.1	4 340	870	Carbon	20.0	19.6
Portsmouth, N. H.	Highway	Lift	28	1	6	297.1	4 660	996	Carbon	21.3‡	19.6
San Mateo, Calif.	Highway	Fixed	27	..	..	297.5	7 620	1 620	Carbon	21.3	20.0
Newark Bay, N. J.	Railway	Lift	Two*	..	..	299.0	26 450	3 700	Silicon	14.0	14.2
Newark Bay, N. J.	Railway	Lift	Two*	..	..	210.8	27 690	2 630	Silicon	9.5	9.5
Bath, Me.	Railway-highway	Fixed	One* 20	..	..	330.0	21 460	4 024	Silicon	18.7	18.6
Lexington, Mo.	Highway	Fixed	20	..	..	408.0	6,952	2 040	Carbon	29.3	29.3
Lexington, Mo.	Highway	Fixed	20	..	..	246.0	6 290	1 092	Carbon	17.3	17.0
Cape Cod Canal, Mass.	Railway	Lift	One*	..	..	544.0	14 920	4 880	Silicon	32.7	32.2
Charleston, S. C.	Highway	Cantilever	20	..	..	640.0	6 284‡	1 726‡	Silicon	27.5‡	27.2
Charleston, S. C.	Highway	Cantilever	20	..	..	1 050.0	7 841‡	2 971‡	Silicon	37.9‡	37.0

* Number of tracks indicated for railway bridges.
† Average for two trusses of different live loadings in the same span.
‡ See reference to this item in the text.

The large variation recorded for the Springfield (Mass.) Bridge (Column (11), Table 2) is due to three causes:

(*a*) The bridge is a deck-span structure;
(*b*) The truss depth is only seven-tenths of the economic depth; and,
(*c*) The top chords of the four trusses carry, in bending, a portion of the flooring and its live load, each chord acting as one of the stringers.

Referring to the Charleston (S. C.) Bridges, all the values in Columns (8), (9), and (11), Table 2, were determined by the method previously outlined. The total loads, the weights of metal in trusses, and the computed "percentage ratios" are the averages for the total length of structure; that is, they cover the trusses of the suspended span, the two cantilever arms, and the two anchor arms of a Type *A* cantilever bridge.

A good idea of the accuracy of the "percentage ratios" given on the diagrams may be obtained from the following analysis of Table 2:

(*A*) The variations, regardless of plus or minus signs, between the computed and the diagrammed ratios, omitting from consideration the Portsmouth lift-span, average 0.4 per cent.

(*B*) The corresponding average of the variations, when the plus and minus signs are duly considered, is 0.3%—showing that, in general, the diagrams "under-run" by that amount. The cause of this under-run, undoubtedly, is that in a few cases the truss weights were somewhat greater than usual,

owing to some slight abnormality in the governing conditions, whereas all the curves were determined, as closely as practicable, for truly economic structures.

When one considers all the causes that may affect the weight of metal in any truss, especially the unavoidable idiosyncrasies of bridge designers, an average variation of less than 0.5% in truss weights is very small, indeed; and an extreme variation (unaccounted for) of less than 1.5% is by no means excessive.

A useful deduction can be drawn from the computations of the 264-ft span of the proposed Jacksonville Bridge. They were made for four cases—carbon steel, silicon steel, ordinary reinforced concrete flooring, and open-grating flooring. The corresponding truss weights were, respectively, 1 200, 900, 930, and 750 lb per lin ft, indicating a saving of metal from the lighter flooring of 22.5% in the trusses of the carbon-steel structure and one of 16.7% in those of the silicon-steel structure. There was also, of course, a further saving of metal in the floor system. From the sum of the money values of these two savings, in any case, must be deducted the extra cost of the flooring itself. In this case the showing favored the open-grating floor. The longer the span the greater is the net saving thereby; and it is evidently much more pronounced in carbon-steel structures than in silicon-steel structures.

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DISCUSSIONS

FLEXIBLE "FIRST-STORY" CONSTRUCTION FOR EARTHQUAKE RESISTANCE

Discussion

BY NORMAN B. GREEN, ESQ.

NORMAN B. GREEN,[14] ESQ. (by letter).[14a]—The active interest in this subject, as evidenced by the discussion, is gratifying. One of the main objects of the paper was to elicit ideas and criticism from other engineers, since the fundamental idea involved is relatively new.

The flexible first story, perhaps, may be considered as an expedient for rendering the structure susceptible of dynamic analysis. The only other alternative is to utilize the so-called "rigid" type of design, in which the entire building is assumed to move with the ground and is subjected to the ground acceleration. This assumption is only permissible if the free vibration period of the structure is very short, which limits this type of construction to low buildings having some very stiff form of bracing, such as masonry walls.

The entire analysis of the building with a flexible base devolves upon the assumption that the upper part of the structure may be considered as a rigid mass. This assumption has been called in question by Mr. Johnson and Mr. Smits. The writer believes that the question is not as to whether the assumption is valid, but rather within what limits it is valid. One extreme would be a wide building of two stories, having concrete second-story walls with a small window area, supported on long first-story steel columns. In this case there seems to be no question but that the assumption is valid. On the other hand, a narrow building of thirty stories, having a large window area, would offer considerable uncertainty. The problem presented by the assumption is not susceptible of theoretical analysis, and the writer is therefore greatly indebted to Mr. Williams for his experimental investigation along this line. Reference to Fig. 13 of Mr. William's discussion will show that the displacement curves for the normal and rigid structure for

NOTE.—The paper by Norman B. Green, Esq., was published in February, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: In May, 1934, by Messrs. Lee H. Johnson, Edward J. Bednarski, and Merit P. White and Paul L. Kartzke; August, 1934, by Howard G. Smits, Esq.; and November, 1934, by H. A. Williams, Assoc. M. Am. Soc. C. E.

[14] Structural Engr., San Francisco, Calif.

[14a] Received by the Secretary January 12, 1935.

Tests Nos. 4 and 8, are very closely the same. This favorable result was secured with a ratio of second-story stiffness to first-story stiffness of only 38.8. The writer believes that a much greater stiffness ratio than this can easily be secured.

Consider, for example, the 20-story steel frame building described by the writer and to which he applied the analysis. The effective height of the flexible first story is 36 ft, which gives an elastic constant of 3 760 000. If it is assumed that the earth motion is parallel to the short dimension of the building, the lateral load acting on the upper or "rigid" part is resisted by the two end walls, each consisting of four 20-ft bays with an over-all width of 82 ft. If there are two 4 by 7-ft windows in each bay, the window area will be 23% of the total, which is a fair average condition for buildings of this type. As an approximation it may be assumed that the lateral distortion in any one story is produced by the shear and bending deflection of the individual piers between the windows, these being taken as fixed at the window head and sill. On this basis, with a 10-in. concrete wall, the elastic constant for one story is 840 000 000 and the ratio of this constant to that for the first story is 223. This computation neglects that part of the story distortion that arises from the bending of the spandrels and the consequent rotation of the ends of the piers. Inasmuch as the spandrels are relatively very deep and short and since about one-half the total deflection of the average pier is due to shear, it is not likely that the error involved is large.

The writer is indebted to Messrs. White and Kartzke for their analysis of the 20-story building, utilizing the acceleration diagram of Fig. 9, in which a condition of resonance is approached. The long waves of this diagram have an acceleration period, not of 3½ sec as they state, but of 3 sec—so that complete resonance with the building having a free period of 3.58 sec is not attained. Even so, an examination of Fig. 10 shows a rapid increase in the building deflection. The important point demonstrated is that only one or two long swings of a period fairly close to the period of the building, are necessary to produce destructive deflections.

Messrs. White and Kartzke are certainly correct in stating that a careful choice of an acceleration pattern will lead to destructive deflections for any given structure; but it is also possible to conceive of a type of gravity loading that will cause the failure of any structure. The occurrence of an acceleration pattern of the character of Fig. 9, having a period of 3 sec and an acceleration of 10% gravity, is not probable in the light of present knowledge of the subject. Referring to the accelerograph records of the Long Beach earthquake of March, 1933,[15] it will be found that there is a well-marked association of long-period waves with low accelerations and short-period waves with high accelerations. The longest waves recorded had a period of about 2.5 sec, with a maximum acceleration of from 1% to 4% gravity; the highest recorded accelerations of from 20 to 30% of gravity, on the other hand, were associated with short periods of from 0.7 to 0.3 sec. If this proves to be characteristic of all earthquakes, the design of so-called

[15] *Engineering News-Record*, June 22, 1933.

"rigid" structures offers serious difficulties, since to be consistent very high accelerations and seismic coefficients should be applied. On the other hand, the flexible-base structure is relatively immune to the effects of these very high short-period accelerations.

As demonstrated by Messrs. White and Kartzke the acceleration diagram of Fig. 3 shows a total ground displacement of about 20 in. The ground motion is periodic up to the end of the fourth second, with an amplitude of motion of 1.6 in. At this point the equilibrium is disturbed by the introduction of an irregularity in the acceleration diagram, so that the ground motion thereafter is of varying velocity, in a constant direction. Obviously, this same diagram could not be extended indefinitely, since it would result in an unlimited ground displacement; however, the progressively increasing ground movement can be halted at any point by the insertion of another appropriate irregularity into the diagram. This simply illustrates the desirability of integrating each acceleration diagram before it is employed in the analysis of a structure. This can be done quite easily by making use of the fact that the algebraic sum of the areas enclosed between the acceleration diagram and the axis of zero acceleration up to any time, is equal to the velocity at that time; the algebraic sum of the areas enclosed between the velocity diagram and the axis of zero velocity gives the displacement during that period of time. The areas can readily be determined by plotting the acceleration diagram and the velocity diagram on cross-section paper.

In this connection the important point is that introduced in the discussion by Mr. Smits, namely, that the insertion of a slight irregularity in the acceleration diagram, which institutes a progressive ground motion in one direction, produces large deflections of the building. No such result can be revealed by the conventional investigation in which only a simple harmonic motion of the ground is considered. The effect is entirely independent of resonance.

The question may arise as to whether any such sustained motion of the ground in one direction is possible. This type of motion cannot be recorded during the progress of the earthquake by any existing seismograph or displacement meter; however, precise surveys made before and after the San Francisco earthquake of 1906[16] indicate large permanent displacements. This is illustrated by Fig. 14 which has been plotted from the results of these surveys. A survey line crossing the main fault at right angles (and which was straight prior to the earthquake) was subsequently found to be broken at the fault, the west half having been displaced 3.6 m toward the north and the east half, 2.3 m toward the south. This lateral displacement of the ground progressively decreased each way from the fault line, as indicated on the diagram, so that a large area of land suffered a permanent shear distortion.

Mr. Smits has extended the writer's equations to include the effect of damping, making this proportional to the relative velocity. Since the lateral load is intended to be resisted in the flexible story by means of steel columns

[16] Rept. of the California State Earthquake Investigation Comm., Vol. 11.

alone and without the help of exterior walls or interior partitions, it does not appear that there would be any considerable damping as long as the stress in these columns is kept within the elastic limit of the material. Further application of the analysis to flexible-base buildings might indicate,

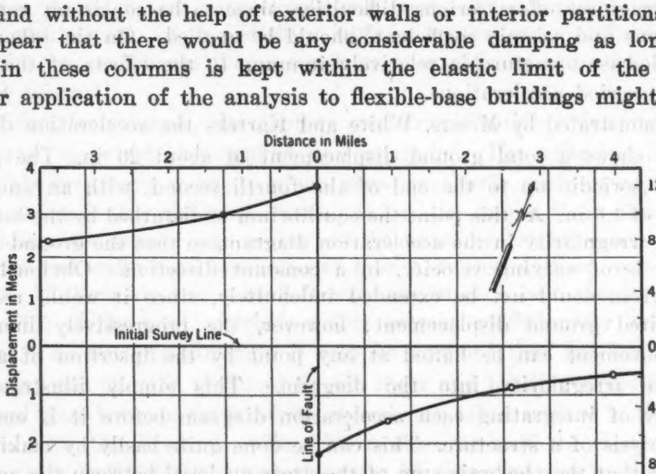

FIG. 14.—PERMANENT GROUND DISPLACEMENTS IN THE SAN FRANCISCO EARTHQUAKE OF 1906

however, the desirability of introducing some form of artificial damping, so as to limit the effects of possible resonance with the ground motion. Perhaps, this could be done by locating the flexible story below the ground, with the base of the rigid superstructure at the first-floor level, so that damping devices could be introduced at this point to re-act against the earth. With this position of the flexible story, all walls and partitions above the ground level could be made rigid and the practical difficulties mentioned by Mr. Williams, attendant upon securing a flexible story in the superstructure, would be eliminated. In this case, the flexible story would also meet the theoretical requirement of a strictly constant value of the elastic factor, *e*, so that there is no uncertainty as to the starting conditions. As stated by Mr. Williams, this will not be the case if it is necessary to rupture first-story walls and partitions before the lateral load is carried by the steel columns in the flexible story.

Mr. Johnson states that the effects of a vibratory phenomenon upon any object cannot be determined by considering only the acceleration. This is not true for non-harmonic vibration of the type the writer has utilized in the analysis, since in this case the acceleration may be established entirely independently of both the amplitude and the frequency. Given an arbitrary acceleration diagram, the first integration establishes the velocity and the second determines the displacements or ground motion. Two constants of integration are introduced, which are known if the initial conditions of velocity and displacement are known. The motion is then completely determined. The statement referred to is correct in the case of simple harmonic motion, the fundamental definition of which establishes a relation between acceleration, amplitude, and frequency.

The discussion by Mr. Johnson mentions an uncertainty regarding the writer's notation for y. This quantity, y, is defined as the deflection of the body from its position at rest; that is, at rest relative to the support. In other words the variable, y, is relative displacement. It would be a clearer statement of the dynamic relation, perhaps, if the writer's Equation (1) had been written in the form:

$$\frac{d^2 y}{d t^2} = a \left(1 - \frac{t}{T}\right) - \frac{e y}{m} \ldots\ldots\ldots\ldots\ldots(37)$$

which is simply an algebraic relation between three accelerations, considered as vector quantities. It states that the acceleration of the mass relative to the support is equal to the absolute acceleration of the support minus the relative acceleration impressed on the mass by the spring. The conventional form of equation representing a forced vibration involves two variables: (1) The absolute displacement of the support; and (2) the absolute displacement of the vibrating mass. The difference between these two variables is the displacement of the mass relative to the support. The writer considers it simpler and more direct in this case to introduce one dependent variable only, which is the relative displacement that it is desired to determine.

The idealized system shown in Fig. 2, in which the vibrating mass is assumed to move only in a straight line in the plane of the paper, has one degree of freedom and only one mode of vibration. The natural period of this vibration is correctly given by Equation (9), which Mr. Bednarski believes to be too simple to be true. It is this very simplicity of dynamic analysis which recommends the flexible-base building. On the other hand the dynamic analysis of a bar fixed at one end and free at the other, cited by Mr. Bednarski, involves harmonics and is a much more involved problem. Such a cantilever beam is similar, in general, to a high building that is flexible throughout its height, and well illustrates the uncertainties and difficulties attendant upon any dynamic analysis of such a structure.

An actual flexible-base building has two degrees of freedom for motions of translation; however, the columns should be arranged so as to give the same elastic constant about either axis, so that its natural frequency will be the same for vibrations in either direction. In case the center of mass of the structure does not coincide with the center of elastic resistance, a couple is introduced, which will produce rotatory vibrations superimposed upon those of translation. This case is outside the scope of the writer's paper.

No mention has heretofore been made of the seismic forces acting on the upper or "rigid" parts of the structure; but these forces must be considered in designing the bracing system, since the requisite strength as well as stiffness must be secured. The acceleration of the upper part of the building is produced by the horizontal reaction of the top of the base columns and, therefore, it will be a maximum when the deflection of the base columns is a maximum. Then, if a_m is the maximum acceleration to which the upper part of the building is subjected:

$$a_m = \frac{e \times \text{maximum column deflection}}{m} \ldots\ldots\ldots\ldots(38)$$

Consider, for example, the 20-story building referred to under the heading "Application of the Analysis" in the paper. The mass of the rigid part of this building is 1 220 000; the elastic constant, e, is 3 760 000; and, from Table 1, the maximum column deflection is 0.445 ft. Substituting these values in Equation (38), the acceleration is found to be 1.37 ft per sec per sec. The maximum deflection that occurs during the first 4 sec of periodic ground motion, is 0.145 ft which gives to the building superstructure an acceleration of 0.45 ft per sec per sec. This is only 14% of the full ground acceleration and illustrates the large reduction in earthquake stresses that may be produced by a flexible story when the ground motion is periodic and even if it is of fairly long period.

In conclusion, the writer would like to express his appreciation of the constructive discussion and criticism that has been brought forth by this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

STRESSES IN SPACE STRUCTURES

Discussion

By F. H. Constant, M. Am. Soc. C. E.

F. H. Constant,[9] M. Am. Soc. C. E. (by letter).[9a]—One of the discussers, Mr. Osgood, has offered a simplified presentation of the derivation given in the paper. It may be simplified still further (avoiding reference to a distant axis) as follows: The force, F, is resolved into horizontal and vertical components:

$$F' = F \cos \phi \quad \text{.......................} (17)$$

and,

$$R = F \sin \phi \quad \text{.......................} (18)$$

both acting at O in Fig. 1. At some point in the horizontal plane through O let two equal and opposite forces act, each equal and parallel to F' and distant, b, from it. Then F is reducible to: (1) The vertical force, R, acting at O; (2) a horizontal (or conjugate) force, equal, parallel to, and acting in, the same direction as F', but distant, b, from it in the horizontal plane through O; and (3) a horizontal couple:

$$S_o = F' b \quad \text{.......................} (19)$$

in which, either S_o or b may be chosen at will. From Equations (17), (18), and (19),

$$b = \frac{S_o}{R} \tan \phi \quad \text{.......................} (20)$$

If a group of concurrent forces acting at O is in equilibrium, and if each force of the group is in like manner resolved, it is immediately clear that $\Sigma R = 0$, and the vector sum of the conjugate components is also zero. These latter are in equilibrium, then, against translation.

NOTE.—The paper by F. H. Constant, M. Am. Soc. C. E., was published in May, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1934, by Messrs. William O. Osgood, L. E. Grinter, and Charles M. Spofford; and October, 1934, by A. H. Finlay, Assoc. M. Am. Soc. C. E.

[9] Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

[9a] Received by the Secretary, January 23, 1935.

It can also be shown that the couples are in equilibrium, namely, $\Sigma S_0 = 0$, provided S_0 for each force, F, is properly selected. Let it be so chosen, in each case, that the ratio, $\frac{S_0}{R} = a =$ a constant. Then Equation (20) becomes identical with Equation (4). From Equations (4), (17), and (18),

$$\Sigma S_0 = \Sigma F' b = a \Sigma F' \tan \phi = a \Sigma R = 0 \dots\dots\dots\dots (21)$$

Hence, the couples, S_0, are in equilibrium. Since the conjugates have numerically the same moment about O as the couples, but of opposite sign, they too are in equilibrium against rotation. The conjugates, therefore, constitute a system of non-current planar forces in equilibrium. Each conjugate is located by the equation, $b = a \tan \phi$, in which a is any arbitrarily chosen constant, expressed in feet. With this method of analysis the conjugates have the same direction as the horizontal components of the F-forces.

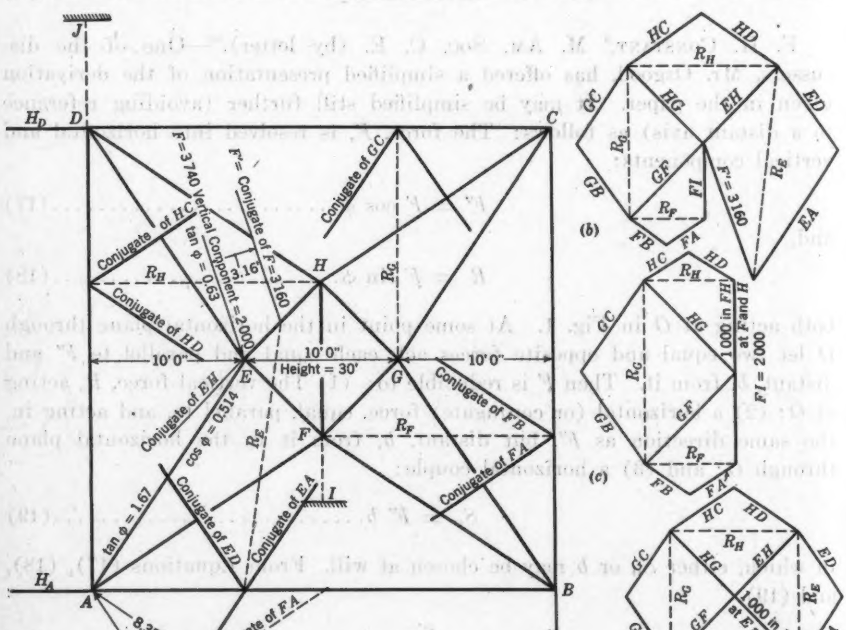

FIG. 8.

Professor Grinter's problem (Fig. 4) presents a good opportunity for demonstrating the relative simplicity of the conjugate method, although Professor Grinter has not done full justice to his own case. By the use of substitute members, the number of simultaneous equations could have been reduced to two, as is shown in the following solution by conjugate stresses (see Fig. 8).

Fig. 8(*a*) is a reproduction of Professor Grinter's Fig. 6, with all the necessary conjugates shown. The members, *E F* and *H F*, are removed and substitute horizontal members, *F I* and *D J*, inserted at joints where the analysis shows that they are obviously needed. Thus, as the joints are taken up in the order *E*, *H*, and *G*, the stress in *G F* becomes known, and it is obvious that *F A* and *F B* cannot hold *G H* in equilibrium. The substitute member, *F I*, therefore, is inserted. The procedure will show that none of the other joints offers any difficulty until *D* is reached, where a second substitute member, *D J*, is necessary. The solution involves three steps: Fig. 8(*b*) shows the conjugate stresses due to the external force at *E* (the members, *E F* and *H F*, being removed); Fig. 8(*d*) shows the conjugate stresses due to a pair of equal and opposite unit loads, acting at *E* and *F* in the direction of *E F* (the external load and *H F* being removed); and, Fig. 8(*c*) shows the conjugate stresses due to a pair of equal and opposite unit loads, acting at *H* and *F* in the direction of *H F* (the external load and *E F* being removed). An inspection of these three polygons shows how simple and symmetrical they are, and how quickly the stresses may be written.

The stresses from these three cases, indicated by S_a, S_b, and S_c, respectively, are listed in Table 2. Those in the lower horizontal members and in the

TABLE 2.—STRESSES IN THE PEDESTAL IN FIG. 4 (SEE, ALSO, FIG. 8)

(Tensile stresses are positive; vertical reactions are positive when acting upward; and, horizontal reactions are positive when acting away from the joint)

Member (1)	S_a (2)	S_b (3)	S_c (4)	Conjugate force, F' (5)	Cosine ϕ (6)	Actual force, F (7)	Member (1)	S_a (2)	S_b (3)	S_c (4)	Conjugate force, F' (5)	Cosine ϕ (6)	Actual force, F (7)
E H...	−2 336	−1 000	....	+ 114	1.00	+ 114	*A D*...	+3 391	+1 178	− 333	+ 230	1.00	+ 230
H G...	+2 336	+1 000	−1 414	−1 280	1.00	−1 280	*D C*...	+2 750	+1 178	−1 167	−1 100	1.00	−1 100
G F..	−2 336	−1 000	+1 414	+1 280	1.00	+1 280	*C B*...	+2 750	+1 178	−1 333	−1 236	1.00	−1 236
F E...	....	+1 000	....	−2 450	1.00	−2 450	*B A*...	−1 933	−1 178	+1 167	+1 916	1.00	+1 916
H F...	....	....	+1 000	+ 825	1.00	+ 825							
							H_A...	−4 656	−2 356	+1 667	+2 491	1.00	+2 491
E A...	−3 400	− 848	....	−1 322	0.514	−2 572	H_D...	+5 622	+2 356	−1 667	−1 525	1.00	−1 525
E D...	+2 200	+ 848	....	+ 122	0.514	+ 237	H_B...	+4 956	+2 356	−2 667	−3 000	1.00	−3 000
H D...	+1 980	+ 848	− 600	− 593	0.514	−1 154							
H C...	−1 980	− 848	+ 600	+ 593	0.514	+1 154	V_A...	+7 348	+2 834	−1 000	....	....	− 420
G C..	−1 980	− 848	+1 200	+1 090	0.514	+2 120	V_B...	−4 976	−2 834	+3 000	....	....	+4 440
G B..	+1 980	+ 848	−1 200	−1 090	0.514	−2 120	V_C...	+6 612	+2 834	−3 000	....	....	−2 800
F B..	+1 000	+ 848	− 600	−1 575	0.514	−3 065	V_D...	−6 980	−2 834	+1 000	....	....	+ 787
F A...	−1 000	− 848	+ 600	+1 575	0.514	+3 065							
......	....	....	....	....	....	....	*F I*...	+1 650	....	−2 000	....	1.00	....
......	....	....	....	....	....	....	*D J*...	+6 324	+2 356	− 667	....	1.00	....

reactions, including those in the false member, *D J*, are included in Table 2, but are not shown on Fig. 8, as they are most easily obtained by analytic processes in the manner explained in the paper. Since the actual stress in *F I* and *D J* is zero, the following two equations can be written:

$$F\,I = 1\,650 - C \times 2\,000 = 0 \quad \text{(22)}$$

and,

$$D\,J = 6\,324 + B \times 2\,356 - C \times 667 = 0 \quad \text{(23)}$$

Whence, $C = + 0.825$, and $B = - 2.45$. The true conjugate stress in any member is then obtained from the general expression,

$$S = S_a - 2.45\, S_b + 0.825\, S_c \qquad (24)$$

These values are entered in Column (5), Table 2, and the actual stresses, obtained by dividing each conjugate by cos ϕ, in Column (7). They check very closely with those computed by Professor Grinter (see Fig. 5).

To study the different types of space structures and the proper arrangement of the reactions is interesting. Probably some types are distinctly more efficient and economical than others; but as yet very little research has been done in this field. German writers have simplified the equations used in the analytic solution of domes and towers, until the process is reduced almost to a mechanical one; and any one familiar with these processes will naturally regard them with favor. Professor Spofford was the first to present an adequate treatement of space structures in American literature.[7] In his discussion, he expresses a preference for analytical methods of stress determination, but the first choice which the designer of a space structure must make is whether to work with concurrent forces lying in three dimensions of space, or with non-concurrent forces lying in two dimensions. In the former case, he must adopt analytic methods of solution, the graphical method being entirely too cumbersome. With the latter method he may use either, as in the case of any planar structure. In the paper some of the stresses (the lower ring and the reactions) were obtained analytically by the use of the three static equations.

The writer purposely chose a simple form of dome in order to illustrate the principles, but the method applies equally well to domes or towers with more than four legs and with several stories. After the diagonal and post stresses of the upper story have been found, they become external forces applied at the top joints of the second story. The positions of their conjugates will already have been drawn, and the magnitudes of these conjugate stresses will be known. These conjugate stresses may be combined into a single resultant if desired. The solution of the second story then becomes a repetition of that of the first.

The writer appreciates Professor Finlay's gracious remarks and is grateful for his contribution to the simplification of the derivations. His analysis follows closely that of Mr. Osgood, which has already been covered in the earlier part of this closing discussion. No further simplification is possible, and it is probable that this derivation will be preferred to the more complex one of the paper. Professor Finlay's proof that the conjugates of forces that lie in the same plane must intersect in a point, is interesting and clear.

As the writer's sole aim was to make available in English a unique and valuable method of solving space structures, he gratefully acknowledges the contribution which each of the discussers has made in the furtherance of this purpose.

[7] "Theory of Structures," by Charles M. Spofford, M. Am. Soc. C. E., Third Edition. McGraw-Hill Book Co., N. Y., p. 457.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

EXPERIMENTS WITH CONCRETE IN TORSION

Discussion

BY PAUL ANDERSEN, ASSOC. M. AM. SOC. C. E.

PAUL ANDERSEN,[29] Assoc. M. Am. Soc. C. E. (by letter).[29a]—In closing the discussion of this paper the writer wishes to present, very briefly, some results of tests with concrete subjected to long-time torsional moments which, it is to be hoped, will throw additional light on the subject.

The aim of these experiments was to determine the rate of increase (if any) in shearing distortions under continued loading and the influence of aggregate and of different intensities of stress. The shape and general dimensions of the test pieces were those of Type *R* in Fig. 4, except that these flow specimens were made hollow with an inside diameter of 5 in. In such a section, the variation in magnitude of shearing stress will be comparatively small.

In testing, the shafts were arranged in rows of four, and adjacent square parts were firmly bolted together by structural steel clamps. One end of this row was fixed against rotation, and a torsional moment was applied at the opposite end by a lever arm and a concrete block of known weight, as indicated in Fig. 17. The loaded end as well as the joints was supported on 2-in. steel balls, which were greased to allow rotation about the axis of the shaft. The method of measuring angular distortions was that previously explained in connection with Fig. 5.

The recorded data from these tests showed that, due to shearing stresses, mortars and concretes flow in much the same manner as when subjected to compressive forces. The results obtained from two mortar specimens are plotted in Fig. 18. It should be noted that the stresses indicated are those midway between the inner and the outer surfaces, computed from the applied torsional moment. It appears from these tests that the sustained modulus of elasticity in shear is considerably lower than the immediate modulus, as

NOTE.—The paper by Paul Andersen, Assoc. M. Am. Soc. C. E., was published in May, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1934, by E. Mirabelli, Assoc. M. Soc. C. E.; October, 1934, by Messrs. Frank M. Russell, and Leslie Turner; December, 1934, by A. W. Fischer, Esq.; and January, 1935, by Messrs. H. J. Gilkey, and A. A. Eremin.

[29] Instr. in Structural Eng., Coll. of Eng. and Architecture, Univ. of Minnesota, Minneapolis, Minn.

[29a] Received by the Secretary January 23, 1935.

listed in Table 2. The writer concludes from this that the spiral reinforcement in a concrete member under torsion will actually provide a higher percentage of the total torsional resistance than Professor Mirabelli's discussion would indicate.

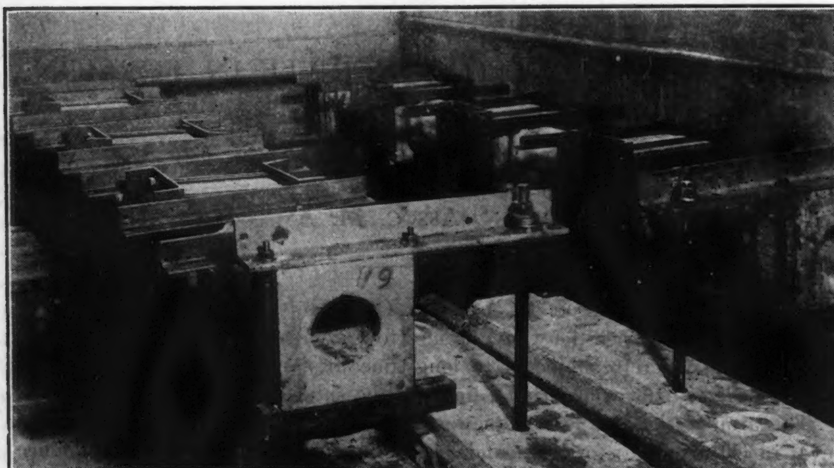

FIG. 17.—CONCRETE SPECIMENS SUSTAINING LONG-TIME TORSIONAL METHODS.

The writer apologizes for not including the Tokyo experiments[12] in the list of previous tests; recently, he has also read with much interest the paper by Messrs. Leslie Turner and V. C. Davies.[13] He agrees with Mr. Turner

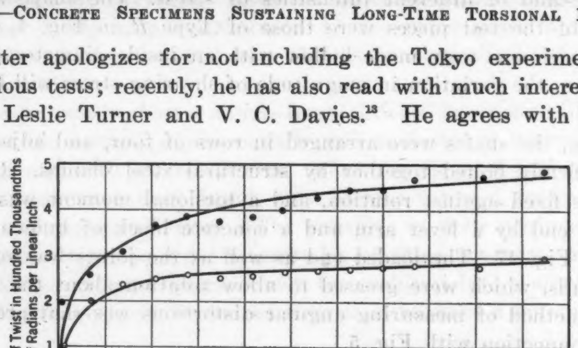

FIG. 18.—TIME SHEAR FLOW CURVES.

as to the difficulty of treating, mathematically, sections of rectangular, T, or L-shape. The empirical formulas referred to by Mr. Turner and fully explained in his paper[13] are interesting indeed.

Professor Gilkey has presented a valuable contribution to the discussion and the writer is pleased to note that the additional data appear to parallel his

[12] "Torsional Strength of Reinforced Concrete," by Miyamoto Takenosuke, *Concrete and Constructional Engineering*, Vol. 22, 1927.

[13] "Plain and Reinforced Concrete in Torsion," by L. Turner and V. C. Davies, *Selected Paper No. 165*, Inst. C. E., London, England, 1934.

own findings. The test results presented by Mr. Russell are interesting, but the writer considers the empirical formulas, Equations (32) and (33), of doubtful value when compared to the established relationship,

$$E' = \frac{E}{2(1 + \sigma)} \qquad (44)$$

between the modulus of elasticity in compression and the modulus of elasticity in shear, σ being Poisson's ratio.

Properly, Mr. Fischer states that most concrete beams transmitting torsional moments are rectangular in section and that the writer's analysis, therefore, cannot be applied directly. In this connection, the writer is much interested in the possible extension of his theory, as presented by Mr. Eremin. Future investigations should be planned to verify Mr. Eremin's equations.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

WAVE PRESSURES ON SEA-WALLS AND BREAKWATERS

Discussion

BY T. L. CONDRON AND CHESTER L. POST, MEMBERS, AM. SOC. C. E.

T. L. CONDRON[19] AND CHESTER L. POST[20], MEMBERS, AM. SOC. C. E. (by letter)[20a].—A helpful service to those members of the profession called upon to design or investigate marine structures has been rendered by Mr. Molitor in his paper. So much has been gathered from widely scattered and often inaccessible sources and has been presented so clearly and concisely, that problems now coming under this title may be more readily investigated. Particularly is this true, since Captain Gaillard's paper[2] is out of print and hence no longer easily obtainable.

The first part of the paper presents the several steps to be taken in determining the wave-pressure curve based on a known fetch and a definite wind velocity. From this wave-pressure curve the theoretical effect of the wave on the structure is determined. The author gives the complete solution of such problems and then applies it to two actual structures, one at Harbor Beach, Mich., the other at Magann's Pier, Toronto, Ont., Canada.

In order to aid in establishing the dependence that may be placed on the author's recommended method, the writers have applied this method of calculating the wave force to two breakwater structures, recently investigated, and reported upon by them. The first is a rock-filled timber crib breakwater, built about 1905, and the second, a proposed rock-filled, steel, sheet-pile cellular type of breakwater, the actual construction of which has since been completed for a length of about 1 500 ft. The latter structure, a continuation of the former, is to be 5 025 ft long and, being in the same line, is exposed to the same intensity of wave action. These two structures are at Calumet Harbor, Ill., near the south end of Lake Michigan.

NOTE.—The paper by David A. Molitor, M. Am. Soc. C. E., was published in May, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: September, 1934, by Charles E. Fowler, Esq.; December, 1934, by Charles T. Leeds, M. Am. Soc. C. E.; and January, 1935, by A. A. Eremin, Assoc. M. Am. Soc. C. E.

[19] Cons. Engr. (Condron & Post), Chicago, Ill.

[20] Cons. Engr. (Condron & Post), Chicago, Ill.

[20a] Received by the Secretary, January 14, 1935.

[2] "Wave Action in Relation to Engineering Structures," by Capt. D. D. Gaillard, Corps of Engrs., U. S. A., *Professional Papers No. 31*, U. S. Corps of Engrs., 1904.

Through the courtesy of Lieut. Col. Donald H. Connolly, Corps of Engineers, U. S. A., M. Am. Soc. C. E., District Engineer of Chicago, Ill., the writers are able to offer the results of their investigation of the designs of these two structures. If the theoretical analysis, outlined by the author, leads to findings in conformity with the actual behavior of the timber structure and indicates a greater factor of safety for the steel structure than for the timber structure, then as he states (see heading "Stability of Breakwater Cribs Subjected to Wave Force"), "the speculative features involved in the method will have been reduced to a negligible quantity."

Following the author's method of investigating the breakwater at Harbor Beach, illustrated by Fig. 6, the writers have determined the various factors for both the Calumet Harbor structures, most of which factors are shown in Figs. 9 and 10. Those not shown there, and the results of the calculations made, are given in Table 6.

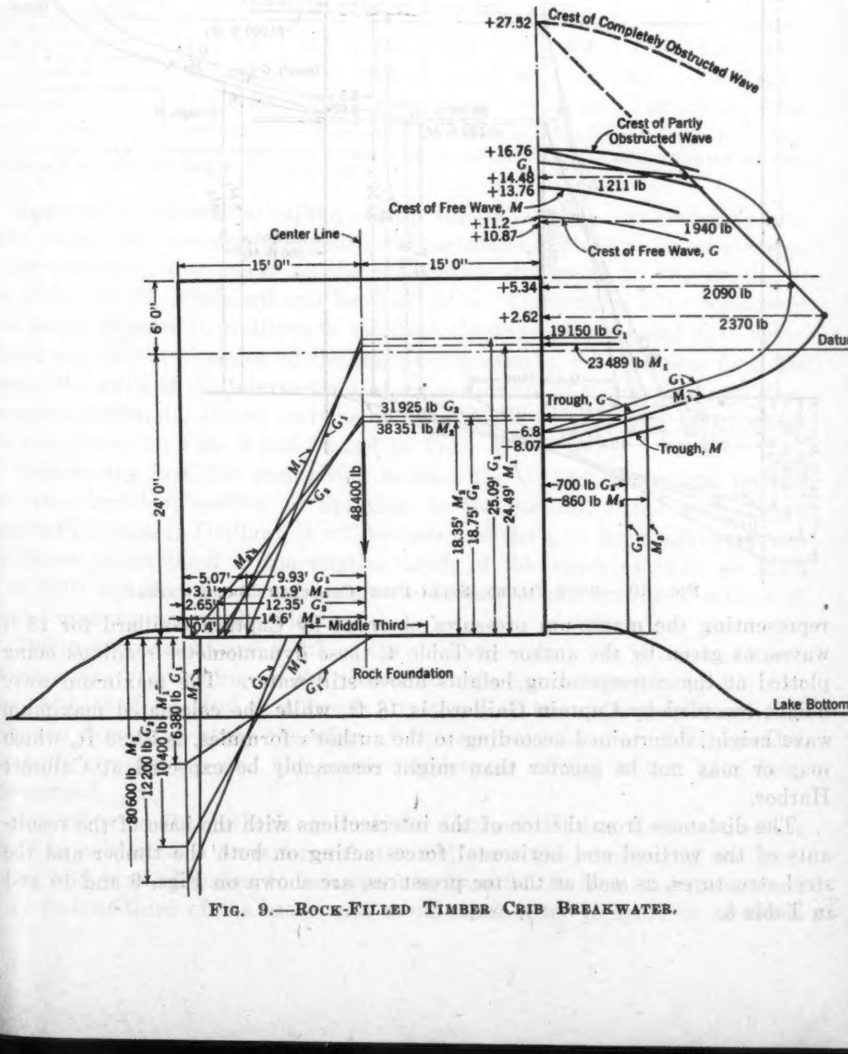

FIG. 9.—ROCK-FILLED TIMBER CRIB BREAKWATER.

In order to make a comparison with the foregoing results, the writers have also determined a wave-pressure curve by plotting on these same diagrams (Figs. 9 and 10), three points, the ordinates from the faces of the walls

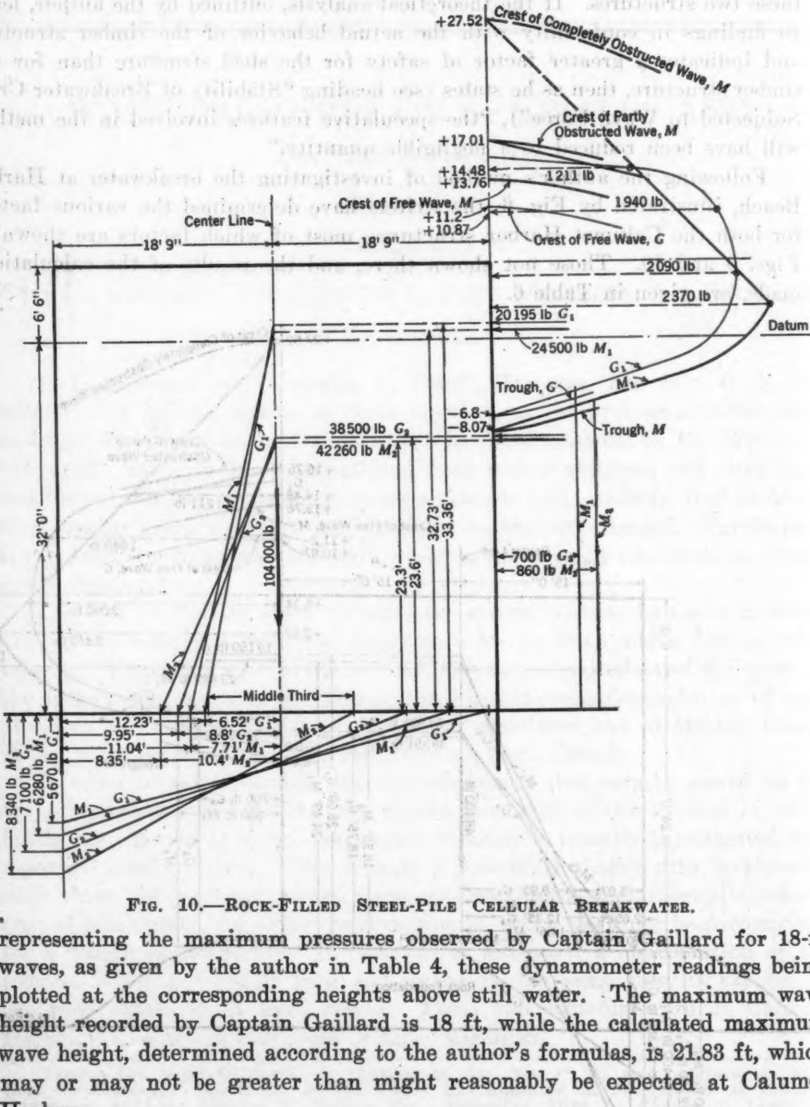

FIG. 10.—ROCK-FILLED STEEL-PILE CELLULAR BREAKWATER.

representing the maximum pressures observed by Captain Gaillard for 18-ft waves, as given by the author in Table 4, these dynamometer readings being plotted at the corresponding heights above still water. The maximum wave height recorded by Captain Gaillard is 18 ft, while the calculated maximum wave height, determined according to the author's formulas, is 21.83 ft, which may or may not be greater than might reasonably be expected at Calumet Harbor.

The distances from the toe of the intersections with the base of the resultants of the vertical and horizontal forces acting on both the timber and the steel structures, as well as the toe pressures, are shown on Figs. 9 and 10 and in Table 6.

TABLE 6.—COMPARISON OF FORCES ON A BREAKWATER

(D = 300 miles, northeast; angle, 60°; V = 55 miles per hr, northeast; and L = 334 ft)

Description	(a) WAVE FORCE ONLY BASED ON:					(b) WAVE FORCE PLUS HYDROSTATIC PRESSURE BASED ON:				
	Notation	Author's Formulas for Structure of:		Gaillard's Recorded Pressures for Structure of:		Notation	Author's Formulas for Structure of:		Gaillard's Recorded Pressures for Structure of:	
		Timber	Steel	Timber	Steel		Timber	Steel	Timber	Steel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Width of base, in feet......	B	30	37.5	30	37.5		......	......	......	......
Weight, in pounds per linear foot............	W	48 400	104 000	48 400	104 000		......	......	......	......
Horizontal pressure, in pounds per linear foot*..	P	23 489	24 500	19 150	20 195	P'	38 351	46 260	31 925	38 500
Height of pressure, P, above base, in feet......	X	24.49	32.73	25.09	33.36	X'	18.35	23.30	18.75	23.60
Distance from toe of structure to intersections of the resultant of W and P with the base, in feet....	Y	3.1	11.4	5.07	12.23	Y'	0.40	8.35	2.65	9.95
Ratio of Y to B expressed as percentages..........	R	10.3	29.4	16.9	32.6	R'	1.33	22.3	8.83	26.6
Toe pressure, in pounds per square foot............	T	10 400	6 230	6 380	5 670	T'	80 600	8 340	12 200	7 100

* The wave force, P, is taken as the force normal to the face of the breakwater considering the direction of the wave as 70° to the face of the breakwater.

Apparently, neither the author, nor the earlier investigators quoted by him, have taken into account the additional unbalanced pressure on the breakwater structure, due solely to the momentary difference in the depths of the water on the windward and leeward sides. The writers have considered this static pressure in addition to the dynamic wave pressure and have determined the centers of action of the combined pressures, the distances from the toes of the walls of the intersections of the resultants of the vertical and these increased horizontal forces, and the corresponding toe pressures, all of which are also shown on Figs. 9 and 10 and in Table 6.

Considering first, the comparison between the author's theoretical method and the empirical method of applying to the structures the wave forces reported by Captain Gaillard, it will be seen that the total horizontal pressures are about proportional to the relative depth of the waves (that is, as 21.83 is to 18.0) and that the elevations above the base of the centers of actions of these forces are practically identical.

The greater weight and width of the steel structure gives it twice as much resistance to overturning as the timber structure, notwithstanding the fact that the base of the steel structure is 8 ft deeper in the water than the timber crib. In this entire study the effect of rip-rap protection (which is provided for both structures), and the passive resistance of the water, have been disregarded.

According to the author's method, the Calumet Harbor timber structure would be subject to a wave force that would cause the resultant of the horizontal and vertical forces to intersect the base 3.1 ft from the toe instead of 10 ft (one-third of the base), and would cause a toe pressure of 10 400 lb

per sq ft. on the rock-fill, which, in turn, is supported by the clay of the lake bottom. According to the Gaillard observations the resultant from an 18-ft wave would intersect the base at 5.07 ft from the toe and result in a toe pressure of 6 380 lb per sq ft.

This timber crib structure has withstood successfully the storms of the past thirty years. In the great storm of 1929, for example the lake level was abnormally high and a northeast wind blew almost continuously for two days, attaining velocities of 50 to 55 miles per hr for several hours duration. It is improbable that the waves of that storm produced as great wave pressure as that determined for this structure by Mr. Molitor's method.

When the author's method is applied to the steel cellular structure at Calumet Harbor, the resultant of the horizontal and vertical forces intersects the base 11.4 ft from the toe, or 0.6 ft outside the middle third, and results in a toe pressure of 6 230 lb per sq ft on the clay bottom of the lake. Applying the wave force based on Gaillard's observations, the resultant intersects the base 0.23 ft inside the middle third and the toe pressure is 5 670 lb per sq ft.

Certainly the author's method appears from this study to be conservative and to give results that may be safely depended upon for a structure designed to be built for the harbors of Lake Michigan and the other Great Lakes.

The addition of the effect of difference in hydrostatic pressure used by the writers in this study would appear to be an unnecessary addition to the formulas of the paper, but certainly it introduces a factor that must be present during wave action, and if the formulas for wave action truly evaluated that force, the factor of the additional hydrostatic pressure would have to be taken into account. It is only because of the probability that the author's formulas err on the side of safety that it may be unnecessary to add the hydrostatic factor.

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DISCUSSIONS

DETERMINATION OF TRAPEZOIDAL PROFILES FOR RETAINING WALLS

Discussion

BY M. A. DRUCKER, ESQ.

M. A. DRUCKER,[6] ESQ. (by letter).[6a]—The analysis of the design of trapezoidal retaining walls, including diagrams, presented by the author, should greatly facilitate the design of such walls. However, the illustrative example solved in Section (9), if followed by others, would lead to irrational designs. In this example the necessary width of base is determined for a wall 20 ft high to support a fill having a sloping surcharge of 30°, whereas the angle of repose is assumed to be 45 degrees. With an assumed top width of 2 ft, the width of base is found to be 6 ft, in order that the resultant shall intersect the base at the outer third point.

It would seem reasonable to assume that a wall designed to resist the pressure of a fill having a 30° surcharge should be capable of withstanding the pressure due to a level fill of the same soil; the factor of safety for the latter case being at least as large as for the former. However, assuming the same wall section, with a 6-ft base and 2-ft top width, and all other conditions being the same as given by the author, the results, as indicated on Fig. 11, were obtained for a level fill. The resultant was found to intersect the base at a distance of 4.62 ft from the back of the wall, which is considerably outside the middle third of the base. As the center of gravity of the wall is 2.16 from the back of the wall, the factor of safety against rotation for this condition would be equal only to $\dfrac{6.0 - 2.16}{4.62 - 2.16} = \dfrac{3.84}{2.46} = 1.56.$ Fig. 11 also indicates the pressure and resultant for the author's design for the condition of a 30° surcharge.

It is not intended to imply that the author's analysis or diagrams are in error; the same results would be obtained by any method using the Rankine theory. Such irrational results are due to, and are inherent in,

NOTE.—The paper by A. J. Sutton Pippard, M. Am. Soc. C. E., was published in August, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1934, by Messrs. E. S. Lindley, and Kenneth L. DeBlois.

[6] Designer in Chg., Board of Transportation, New York, N. Y.

[6a] Received by the Secretary January 2, 1935.

the Rankine assumption that the direction of the resultant earth pressure is parallel to the ground surface. In accordance with this method, while the

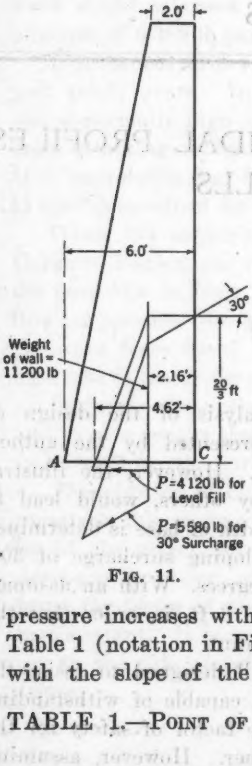

FIG. 11.

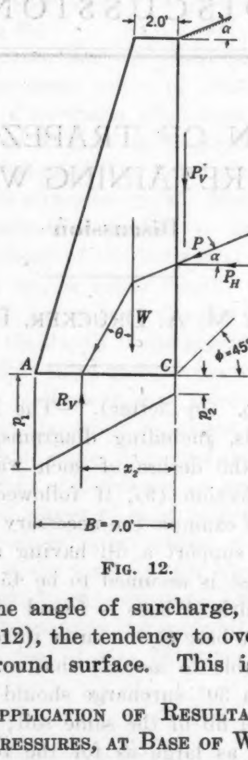

FIG. 12.

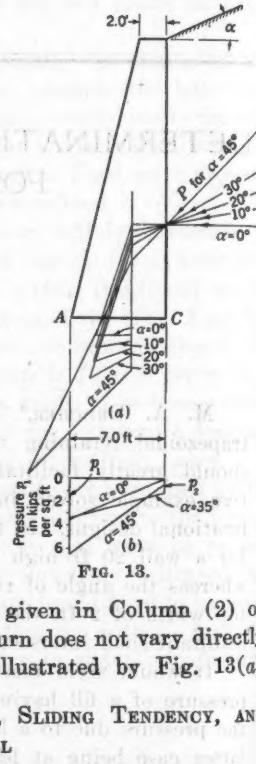

FIG. 13.

pressure increases with the angle of surcharge, as given in Column (2) of Table 1 (notation in Fig. 12), the tendency to overturn does not vary directly with the slope of the ground surface. This is illustrated by Fig. 13(*a*)

TABLE 1.—POINT OF APPLICATION OF RESULTANT, SLIDING TENDENCY, AND PRESSURES, AT BASE OF WALL

Angle of surcharge, α, in degrees	Applied Pressure, in Pounds			Moment, M_C,* in foot-pounds, of P and W about Point C, Fig. 12	Vertical component of resultant, R_V (= $W + P_V$), in pounds	Distance, X_2 (in feet) $= \frac{M_C}{R_V}$	Intensity of Pressure, in Pounds per Square Foot		Value of tan θ $\left(= \frac{P_H}{R_V}\right)$
	P	P_V	P_H				At toe, P_1	At head, P_2	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0......	4 120	0	4 120	58 800	12 600	4.67	3 600	0	0.328
5......	4 130	360	4 120	58 800	12 960	4.53	3 490	215	0.318
10......	4 260	740	4 190	59 300	13 340	4.45	3 450	350	0.315
15......	4 370	1 130	4 220	59 400	13 730	4.32	3 340	580	0.308
20......	4 610	1 570	4 330	60 200	14 170	4.25	3 330	720	0.306
25......	5 020	2 120	4 550	61 700	14 720	4.19	3 340	860	0.309
30......	5 580	2 790	4 840	63 600	15 390	4.14	3 410	990	0.317
35......	6 480	3 720	5 300	66 700	16 320	4.09	3 510	1 150	0.325
40......	8 170	5 250	6 260	73 100	17 850	4.10	3 860	1 240	0.350
45......	17 000	12 000	12 000	111 300	24 600	4.59	6 630	405	0.488

* M_C = moment of P and weight of wall about Point C.

which shows the points of application, at the base of the wall, of the resultant due to the earth pressure and weight of wall for several angles of surcharge. The wall, 20 ft high, was designed to support a level fill with the

resultant for this condition intersecting the base at the outer third point, the necessary width of base being 7.0 ft.

It may be seen from Fig. 13 that for all other cases, governed by the author's conditions as to weights and angle of repose, including that for the maximum surcharge of 45°, the point of application of the resultant is within the middle-third area of the base of the wall designed for a level fill. This denotes that, as regards rotation, the wall would be in a safer condition for sloping fills than for a level fill. It may be advisable, therefore, when designing a wall for a sloping surcharge, to make sure that the wall would be equally safe in case it be decided later not to place the fill on the slope assumed. Otherwise, one might find himself in the embarrassing position of having to insist either that the fill must be placed at the angle assumed in the design or that the wall should be made wider.

It may also pointed out that the author's analysis and diagrams aim at obtaining a base of such width as to have the resultant pressure intersect it at the outer third point. While such a condition provides for bearing on the entire base, with a maximum pressure at the toe and zero pressure at the heel, it does not give an exact indication of the factor of safety of the wall against rotation because, with such a limitation, this factor would be 2 for a triangular wall and 3 for a wall rectangular in shape. Consequently, for any particular case, the factor of safety against overturning would vary between these limits.

In the design of retaining walls, of course, other conditions are to be considered, in addition to that of rotation treated by the author. One other phase of design that should be considered is that of sliding on the base. The tendency to slide may be measured by the tangent of the angle between the resultant pressure line and the vertical. Fig. 13(*a*) shows that this angle would be larger for the case of a level fill than for a sloping surcharge up to about 35 degrees. This is demonstrated more clearly in Column (10), Table 1.

Another condition that should be investigated is that of maximum pressure on the soil at the toe of the wall. From Fig. 13(*b*), which shows the variation in pressures under the wall, as obtained for a level fill as well as for several angles of surcharge, it is clear that a level fill will cause greater pressures than sloping surcharges up to about 35 degrees. Column (8), Table 1, shows the variation in pressures for various angles of surcharge. It may also be noted that while, as regards overturning, the wall is safer to resist the pressure due to the 45° surcharge than that due to the level fill, the pressure at the toe of the wall is nearly twice as large for the former condition as for the latter. Therefore, a design based only on the stability to resist overturning may not reveal other conditions which should be taken into consideration.

From the preceding it may be seen that, if the magnitude and direction of pressures be obtained by following the Rankine theory, the pressures of sloping surcharges up to an angle of about 75% of the assumed angle of repose, need not be considered if they act against a wall having a vertical back.

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DISCUSSIONS

SECURITY FROM UNDER-SEEPAGE MASONRY DAMS ON EARTH FOUNDATIONS

Discussion

BY MESSRS. W. M. GRIFFITH, AND E. MCKENZIE TAYLOR

W. M. GRIFFITH,[43] ESQ. (by letter).[43a]—In general, the writer agrees with the weighted-creep theory presented in this paper. As the extra stability of the material along the vertical creep must be due, to some extent, to the higher intensity of pressure within the material at the greater depth, the extra stability of the material in the vertical creep will depend, to some extent, on the depth of the vertical creep. This factor should also be considered in applying the theory to special cases. The relative value of the horizontal creep must also depend, to some extent, on the internal pressure in the material in immediate contact with the foundations, and this pressure will be that resulting from the downward pressure of the structure on the material. Its intensity will depend on the design of the structure. As a result of the application of the hydraulic gradient theory, such structures are now designed more lightly than in former years, and, in modern structures, the resultant pressure along the line of horizontal creep is less than in old designs. The relatively higher value of vertical creep, therefore, is intensified in modern structures.

The author has referred to a paper by the writer, on this subject,[8] published in 1913–14. At that time, interlocking steel piling was unknown on irrigation works in the United Provinces (India) and, where used, sheet-piles were of timber, driven by native labor and, consequently, were not especially reliable. Subsequent experience with interlocking steel piling has caused the writer to modify, to some extent, the views expressed in his paper.

In suggesting a reduction of 20% in the length of creep, where reliable vertical sheathing to a depth of 10 ft was used, the writer had in mind a

NOTE.—The paper by E. W. Lane, M. Am. Soc. C. E., was published in September, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1934, by Messrs. William P. Creager, and L. F. Harza; and January, 1935, by Messrs. Joel D. Justin, and Louis E. Ayres.

[43] London, England.

[43a] Received by the Secretary December 27, 1934.

[8] "The Stability of Weir Foundations on Sand and Soil Subject to Hydrostatic Pressure," *Minutes of Proceedings*, Inst. C. E., Vol. 197, Pt. III, 1913–14, p. 221.

single line of piling and dams of the class with which he was then (1913) dealing (which, in general, were founded on fine sand and supporting heads of about 10 ft). The value assumed for the safe creep ratio being about 15, such structures would require a 150-ft length of horizontal creep or, with 20% deduction, a 120-ft length of combined horizontal and vertical creep. The vertical creep for a 10-ft depth of piling being 20 ft this 120-ft length of combined creep would be divided into a 100-ft length of horizontal, and a 20-ft length of vertical, creep. The writer, therefore, was giving the 20-ft length of vertical creep the equivalent of a 50-ft length of horizontal creep—a ratio of $2\frac{1}{2}:1$.

The author's value of this ratio is 3 to 1, but it will be seen from his Table 3 that his safe values for simple horizontal creep (that is, three times his weighted creep) are somewhat higher than those the writer was proposing. As a result of the hydraulic gradient theory, several dams were designed and built in the Rohilkhand Canal Division (United Provinces, India) from about 1905 to 1908, that were founded on fine light sand, relying entirely on horizontal staunching aprons, with no vertical cut-offs. Apparently, these dams were satisfactory for some years but, later, exhibited signs of failure by settlement of some part of the up-stream apron near the crest line. These defects were repaired, but eventually the structures failed completely. Piping was the direct cause of failure, and experience showed that this action can be slow and accumulative. It was generally considered that such action could be detected in time, by observing that springs down stream carried away small particles of sand.

The sudden and complete failure of Kulli Dam by under-seepage, after standing satisfactorily for nine years, showed that this indication could not always be relied upon, as no such indication was observed in this case. In consequence of this experience, the writer would not recommend founding a dam on fine light sand, without one line of vertical piling, designed to carry the line of creep to a depth at which the internal pressure could be relied on to ensure good consolidation.

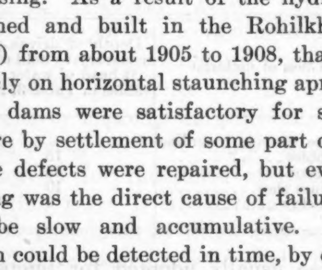

FIG. 2.

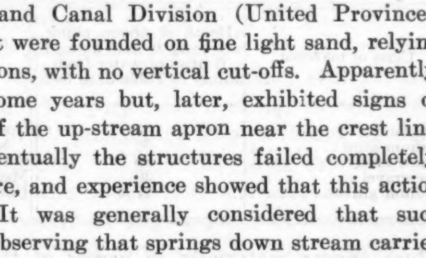

FIG. 3.

The same reasoning does not apply, however, to dams founded on coarse heavy sand, or shingle; the writer has known a number of such structures designed without vertical staunching and has never known a failure from under-seepage.

A good test of Mr. Lane's theory and of his values for the safe weighted-creep ratios, is to consider the extreme case of a dam having only vertical

sheet-piling. The case of a coffer-dam, of interlocking steel piling, driven in the alluvial bed of a river to provide a dry spot for foundation work, affords a common example of this, as depicted in Fig. 2. The length of creep, L, is equal to $2\,d$ and the head is H. Then, the weighted-creep ratio, $\alpha, = \frac{H}{2\,d}$; or,

$$d = \frac{\alpha\,H}{2} \qquad (5)$$

The safe values of d required to resist a 10-ft head of water as calculated from the author's Table 3 for different materials, are given in Table 6. These

TABLE 6.—WEIGHTED-CREEP RATIOS REQUIRED TO RESIST A 10-FOOT HEAD OF WATER

Class of material	Depth of driven piling necessary to hold up a 10-ft head of water from values in Table 3, in feet.	Class of material	Depth of driven piling necessary to hold up a 10-ft head of water from values in Table 3, in feet.
Fine sand	35	Coarse gravel, including cobbles	15
Medium sand	30	Soft clay	15
Coarse sand	25	Medium clay	10
Fine gravel	20	Hard clay	9
Medium gravel	17.5		

values appear materially in excess of those usually adopted for temporary structures; and they seem to be safe values, and to support the author's theory.

The writer does not agree with Mr. Lane's recommendation that two lines of sheet-piling are preferable to one deep line (see heading "Recommended Values Are Conservative"), believing that one deep line, if correctly placed, is more reliable. Interlocking steel piling forms a reliable vertical cut-off and one deep line placed under the crest or line of gates gives the maximum security for the material used.

To illustrate this point consider the case of a dam having a horizontal floor with a line of piles at the up-stream end and down-stream end, as sketched in Fig. 3. From Point A to Point B the downward pressure on the apron is greater than the uplift in the line of creep, and this difference is intensified by the piles at Point A. If this apron is a thin one (as is usual), and if piping or bad consolidation occurs beneath it, it may fail near Point B where the difference in pressure is greatest. Such failure at one point will at once shorten the line of creep by the length of the apron and the line of piles at Point A. This may reduce the creep distance sufficiently to cause a blow-out; in any case it would increase greatly the uplift beneath the floor, BC, and this floor would be likely to blow up under the increased upward pressure, in which case a blow-out would be certain. An initial failure in Apron AB might not be noticed at first because it is submerged. The failure of the Deoha Barrage was undoubtedly an example of this type.

With two lines of piles stability then depends not only on the reliability of both lines, but also on that of the aprons. If the two lines of piling are

placed near together, one on each side of the crest line, *B*, the chance of failure through failure of the apron is reduced, but in that case the short path may prove the critical line of creep, and the value of the two lines is then greatly reduced.

Finally, one line of deep piles will carry the line of creep to an area at which the internal pressure in the material is twice as great as in the case of two lines carried to one-half the depth. Where two lines of piling are used, one being situated at the down-stream end of the apron, the writer would favor the construction of "weep-holes" with inverted filters below in the down-stream apron.

Mr. Lane has presented an interesting and valuable paper; the writer is greatly impressed by the complete data collected, and by the author's method of analysis.

DR. E. MCKENZIE TAYLOR[44] (by letter).[44a]—In considering the percolation path, it is important to distinguish between weirs protected by lines of wells or masonry curtain-walls, upon which Bligh formulated his theory, and those protected by sheet-piles. It is impossible to obtain a perfect seal between the wells or masonry curtain-walls and the permeable material on which the dam is constructed. It follows that in all probability Bligh's theory of creep was correct for this form of construction. When sheet-piles are used for protection there is no disturbance of the foundation material and, hence, the flow postulated by Bligh does not occur. A large number of model experiments[45] have been made to study the lines of flow when a work is protected by sheet-piles, and in no case does the flow occur vertically up the down-stream face of the pile line.

Two instances of weirs confirm the results of the model experiments. Panjnad Weir (Punjab) has three lines of sheet-piles. It has been found possible to reproduce the actual uplift pressures on a model which shows that the pile lines are functioning as they do in the model. The second instance is that of Bay 4, Khanki Weir (Punjab). During reconstruction in 1934, lines of piles were driven. The effect of these piles was tested on a scale model and the results of the actual uplift pressures as observed on the floor of the weir are in close agreement with those obtained in the model. In the data presented in this paper no distinction is made between the two forms of construction. A re-examination of the data taking into consideration the differences in the percolation paths associated with the two methods of construction would be of the greatest value.

The author urges caution in applying the results of model tests, whether obtained by means of the flow net or by the hydraulic electric analogy, to constructional practice. Recent work in connection with a study of Panjnad Weir,[46] has established the following principles:

(*a*) The variation in uplift pressure recorded on the work can be reproduced on a model by varying the deposit of silt up stream.

[44] Director, Irrig. Research, Punjab, Lahore, India.

[44a] Received by the Secretary January 7, 1935.

[45] *Proceedings*, Punjab Eng. Congress, Vol. XXII, 1934, pp. 51–78.

[46] *Loc. cit.*, Vol. XXIII, 1935. (In the Press.)

(*b*) A work designed from the results of a model on a homogeneous permeable foundation will withstand the worst conditions as regards uplift pressure to which it will be subjected.

(*c*) The uplift pressures on a work on a permeable foundation are independent of the grade of the foundation material if it is uniform.

Even under much more complex underground conditions, it has now been shown that, if the conditions are known, they can be reproduced in a model. This was done in the case of Bay 4, of Khanki Weir, in which a clay stratum was found some distance below the sand surface. As a result of these investigations, the results of model experiments were applied to the design of other Bays of Khanki Weir. Since a design based on the results of model experiments deals with the worst conditions that are likely to be experienced, it appears that model work can be applied with confidence.

Under "Relative Weights of Vertical and Horizontal Creep," the author states that "the upward pressure measurements on several structures on which data were available shows that the drop in upward pressure along horizontal concrete surfaces is almost zero." This statement cannot apply to works in a good state of repair. Such a condition in the Punjab would indicate the immediate necessity for investigating the safety of the weir. Two instances of no drop in pressure along the floor have come to the writer's notice recently. In both cases an investigation of the dam showed that flow, as distinct from percolation, was occurring due to the unsound condition of the masonry. In cases where sheet-piling has been used in the Punjab a drop in pressure invariably occurs along the floor.

It is indicated, therefore, that construction which involves the use of wells or masonry curtain-walls is unsound and should be abandoned. In the future, sheet-piling should be used for increasing the percolation path. If this is done, the deductions based on the analysis of data for works in which a line of wells or masonry curtain-walls exist, are likely to be inapplicable. The insertion of pressures pipes and their regular observation is now a feature on some weirs in the Punjab. The importance of inserting pressure pipes in a weir cannot be emphasized too much and the alteration in pressure is a valuable indication of conditions under the floor. Such observations have already enabled action to be taken before the situation became serious.

From his investigation the author concludes that greater weight should be given to the creep along the vertical, than along the horizontal, surfaces. While this may be the case in dams with masonry curtain-walls, the weighting suggested is not borne out either in model experiments or in observations on actual works in which sheet-piles are used. The main drop in pressure occurs around the foot of the pile and is due to the curvature imposed on the flow lines. This drop in pressure is usually four times that experienced on the up-stream face of the pile. The drop in pressure on the down-stream face of the pile amounts to about one-twelfth of that on the up-stream face. It is impossible at present to state any rule for determining the optimum length of pile with reference to the length of floor. The only satisfactory

method at present available for determining the effect of sheet-piles on the uplift pressures, in a complicated structure, is the examination of these piles on a scale model.

A further point of considerable importance in connection with uplift pressures is the "unbalanced head." The normal practice in the Irrigation Research Institute, Punjab, is to examine a model of the work in the flume and plot the water profile for a variety of conditions that are likely to be experienced on the work. From the position of the standing wave for the worst condition and the uplift pressures determined by means of the flow net the unbalanced head can be plotted.

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DISCUSSIONS

AN ASYMMETRIC PROBABILITY FUNCTION

Discussion

BY MESSRS. R. D. GOODRICH, AND F. T. MAVIS

R. D. GOODRICH,[25] M. AM. SOC. C. E. (by letter).[25a]—A notable contribution to the knowledge and literature of asymmetric probability functions is contained in this paper. Little need be added to the discussion, in Section I, of the general probability functions and of the curves in use at the present time. The author has emphasized clearly the practical difficulties and limitations to the use of Pearson's, Thiele's, and the Gram-Charlier series for many engineering problems. They are all mathematically "elegant"; but mathematical elegance has less appeal to the engineer than practicability.

It is well to call attention, as the author has, to the desirability of estimating the reliability of the parameters used in any method. Perhaps still more important, however, is some knowledge of the reliability of the results to be obtained from the final curve or equation. That this seldom is possible with purely empirical methods is quite true.

The writer disagrees with the author's statement to the effect that "the use of * * * and other graphical devices is an undesirable practice." On the contrary, graphical methods are frequently most desirable for the presentation and analysis of engineering and numerous other data. No doubt the author's point is that the value of graphical methods is largely lost if their limitations are not kept clearly in mind. His criticism of the writer's semi-graphical method[12] for the analysis of skew frequency distributions can be met, at least to a large extent, by adapting the method of least squares to the determination of the parameters of the function. These calculations make it possible, with little computation, to determine the probable errors of the parameters and also that of any value of the variable as read from the curve for a given percentage of time.

An interesting graphical method for the application of least squares to the fitting of curves to data has been written by Professor E. O. Waters.[26] The

NOTE.—The paper by J. J. Slade, Esq., was published in October, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: January, 1935, by Messrs. Gordon P. Williams, and H. Alden Foster.

[25] Prof., Civ. Eng., and Acting Dean, Coll. of Eng., Univ. of Wyoming, Laramie, Wyo.

[25a] Received by the Secretary January 2, 1935.

[12] *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 1 *et seq.*

[26] "Graphical Methods for Least Square Problems," *Proceedings*, Applied Mechanics Div., A. S. M. E., 1928.

writer now makes frequent use of an analytical method for Case I of Professor Water's paper, to fit the curves of his method[12] to flood data, with satisfactory results.

Professor Slade has given an excellent statement of the character desirable for an asymmetric probability function, and has presented a form for this function which meets his specifications to a remarkable degree. The solution of the equation involving this function, when partly bounded, in terms of the power sums, or moments of the data, needs no discussion.[26a]

One great advantage of the proposed function is the fact that it includes the normal curve; but a disadvantage is that this curve is a limit as to the skewness in one direction, the skewness always being toward the finite limit of the curve. This is evident from a comparison of Equations (32) and (33) and also from Equation (36), and is stated by the author in the text following Equation (45). An example of flood data for which the mode is greater than the mean, and for which there is no apparent upper limit, is that of the Mackenzie River, in Australia.[27] In this distribution the range of observations above the mean is only 85% of the range below it, although the number of observations above and below the mean is practically the same.

The use of the new equation as an expression of the mathematical frequencies of a skew distribution, as illustrated by the computations for the data of Table 3, involves the computation of ten auxiliary quantities before

TABLE 10.—COMPARISON OF RESULTS, ANALYTICAL AND GRAPHICAL METHODS

Values of the variate, X	Observed frequency	Frequency in percentage of total	Total cumulative frequency (percentages)	GRAPHICAL RESULTS (GOODRICH): Total cumulative frequency (percentages)	Frequency in percentage of total	Frequency number	Difference between observed and graphical frequencies	SLADE: Frequency number	Difference between observed and computed frequencies	PERSON: Frequency number	Difference between observed and computed frequencies
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1....	44	17.6	17.6	14.0	14.0	35	+9	62	−18	59	−15
2....	134	53.6	71.2	72.0	58.0	145	−11	109	+25	111	+23
3....	45	18.0	89.2	89.0	17.0	43	+2	49	−4	45	0
4....	12	4.8	94.0	94.8	5.8	15	−3	19	−7	20	−8
5....	8	3.2	97.2	97.1	2.3	6	+2	8	0	9	−1
6....	3	1.2	98.4	98.2	1.1	3	0	3	0	4	−1
7....	1	0.4	98.8	98.8	0.6	2	−1	2	−1	2	−1
8....	3	1.2	100.0	99.2	0.4	1	+2	1	+2	1	+2
Total..	250	100.0	.....	......	100.0	250	30	.....	57	.....	51
Means.	...	......	.....	......	......	......	3.75	.....	7.13	.....	6.38

the four constants of the equation can be determined. Using skew frequency paper, the writer can plot a curve[12] agreeing more closely with the data, comparatively, with little computation. All that is necessary to plot the diagram is to prepare the first four columns of Table 10.

[26a] **Correct Equation (14) by changing the first minus sign to an equality sign.**

[27] ***Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 20.**

Values of X from Column (1), Table 10, are then plotted against the cumulative values of skew frequency in Column (4) on standard, skew frequency paper, and an average, straight line is drawn by eye through the plotted points. Average values of total frequency are then read from the straight-line curve and entered in Column (5), Table 10. The determination of the data in the remaining columns is obvious. The sum of the differences between the observed and computed frequencies by the graphical method is only three-fifths as great as that resulting from the use of the analytical methods propounded by either Professor Slade or Professor Pearson, while the time and labor required for the graphical solution is only a fraction of that necessary for any analytical solution. Furthermore, the equation of the integral frequency curve requires the determination of only two constants from the diagram, the slope and intercept, which, with the total number of observations, give an equation of the form:

$$t = n\, e^{i} \text{..........................} (51)$$

in which, $i = -k\, x^{-c}$. If the writer's equation involving the lower limit of the frequency curve, $-b$ in the author's notation, is used, and the values of the index and modulus are determined by least squares, very much closer agreement can be obtained between the observed and computed frequencies. Furthermore, the probable errors of these parameters can be determined also. Contrary to the author's statement, the computations for the necessary factors can be made graphically by Professor Water's method, as previously mentioned, although the writer prefers analytical methods for this work. The computation of two means and three other quantities similar to the author's moments, are all the factors necessary for the determination of the two parameters of Equation (51). The computations, therefore, are only about one-half those required by the author's method. The maximum deviation below the mean is not determined statistically by the writer's method. Theoretically, at least, Professor Slade's method has an advantage in this one respect, in that, for the partly bounded function, this quantity can be determined statistically.

It can be demonstrated that the writer's equation can be fitted to a set of observations much closer than by the use of either the author's or Pearson's functions. Furthermore, an equation with four constants, including the number of observations, n, and the lower limit, b, can be fitted without the use of products or moments of the third degree which are required in the author's solution. The use of third or higher moments is a disadvantage, especially for the shorter series of observations, on account of the increase in the magnitude of the errors in the resulting power sums for these higher moments. This fact is recognized and mentioned by the author. The suitability of the value selected for the lower limit of the curve may be tested by use of the coefficient of correlation.

While the duration curve in Fig. 2 approximates the data fairly well, the curve crosses the evident trend of the data in the central part of the figure instead of following it. With the writer's method[12] a curve can be

fitted which has sharper curvature at each end so that the data are more closely approximated by the curve. The question, therefore, arises as to whether or not the new function is sufficiently flexible to become generally useful. The question is one that can only be answered after a great many trials with a great variety of distributions. The rainfall data used show unmistakable evidence of a disturbing influence so that the series is not especially well adapted to illustrate the use of the proposed function. Study of physical conditions might disclose two distinct distributions of storms due to separate sets of causes, which are superposed to form the records given.

The most general function proposed is a logical extension of the partly bounded function and should meet the needs of those who find it necessary to investigate frequency distributions requiring treatment with a more general formula. The writer is of the opinion that extra-statistical considerations are not only justified as an aid in these investigations, but that the physical conditions influencing any phenomena should always be studied, and the results of such studies should be included in and made a part of the investigation. No other method except that of probabilities, is available in many investigations dealing with problems of water supply, river regulation, and flood control, and to pass by the aids offered by the use of such tools as the material in this paper, would appear to be the poorest of judgment on the part of the engineer who does not avail himself of them.

While the writer does not see any great practical advantage in the use of this new function over the semi-graphical methods heretofore proposed, the advantages in many cases over the older functions used for analytical methods are quite obvious. It would be of interest and value to those who contemplate the use of these functions to have the formulas given for the standard errors of the moments and constants necessary for the solution of the equations. The author is to be commended for his able contribution to the literature of this subject.

F. T. MAVIS,[28] ASSOC. M. AM. SOC. C. E. (by letter).[28a]—The mathematics of statistical analysis is likely to become so intriguing that the objectives and the limitations of the analysis may be obscured by algebraic manipulation and integration. Undue attention to analytical formulation may conceivably lead to tacit assumptions and hypotheses and to conclusions which appear to be contradictory. The author states, for example (in the sentence preceding Table 2), that "* * * it must be remembered that appearance, particularly of a graph plotted on logarithmic probability paper, is no test of goodness of fit," and later, in Example *b*, he states "* * * a glance at Fisher's diagram[20] and at Fig. 1 of this paper is sufficient to show that the latter is at least as good a fit." Back of the first statement undoubtedly lies not only the tacit assumption of something like Pearson's test of goodness of fit,[29]

[28] Associate Director, in Chg. of Laboratory, Iowa Inst. of Hydr. Research, and Acting Head, Dept. of Mechanics and Hydraulics, State Univ. of Iowa, Iowa City, Iowa.

[28a] Received by the Secretary January 4, 1935.

[20] "The Mathematical Theory of Probability," by Arne Fisher, 1930, Fig. 4.

[29] "Handbook of Mathematical Statistics," by H. L. Rietz, p. 78.

but also the assumption that the type curve chosen must be expressible in equations of a particular form.

Under "Graphical Methods," the author appears to have blamed the tools rather than the improper or unskillful use of them in his statement that the use of "probability" paper and other graphical devices is an undesirable practice, and that these methods convey to the eye a simplicity which does not really exist. According to Professor Slade, "the process of using these methods is not essentially different from that of fitting a curve to the data, except for the fact that one is more likely to fit the wrong curve graphically than analytically." To one who has had some practice in using both graphical and numerical methods of curve fitting the foregoing dicta are somewhat disturbing, and the writer would be much interested in the arguments supporting the author's statements.

Admittedly, there are cases in which graphical methods are less convenient than numerical methods, and there are border-line cases in which the choice of the one or the other is largely a matter of personal preference, or the whim of the moment. The person equally skilled in the use of both will ordinarily make his choice between them on the basis of: (1) The limits of precision demanded to be consistent with the precision of the given data; (2) the relative amounts of time required to arrive at a solution by the two methods; and (3) the chance taste of the computer at the moment. It is the writer's opinion that the engineer who has frequent occasion to analyze statistical data would do well to acquire skill in the use of both graphical and numerical (or analytical) methods. It is not unlikely that the crudest of methods—graphical or numerical—will satisfy the engineer who at infrequent intervals follows the simplest procedure described in a book which he finds handy.

Any set of observational data to which statistical methods may be applied, must be considered as a sample which may approximate but never precisely define every larger set of the same data. An arrangement showing the frequencies of values in classes arranged in order is called a frequency distribution. For a given sample and given class intervals there is a unique frequency distribution which may be represented graphically by a frequency polygon, or by a histogram which is made up of frequency rectangles. An estimate of the limit that would probably be approached if the class intervals were made smaller and the number of observations larger, without limit, is represented by the frequency curve—a graphical counterpart of the author's frequency functions. It is to this frequency function that the author is directing his analysis, an analysis which is based naturally on the assumption that the variables are continuous instead of discrete.

If due consideration is given to the effects of errors in making observations, and if data are reported with a precision consistent with the probable error of a single observation, all hydrologic variables must be considered discrete rather than continuous variables. If increments of the variables represent a constant numerical difference—as in the case of annual rainfall shown in Fig. 2—linear graduations of the variable appear consistent with

the data. If increments represent constant percentage differences, as might be anticipated in the case of run-off measurements, logarithmic graduations of the variable would appear to be preferred. A wide variety of regular functional co-ordinates can be devised by trial to represent cumulative frequencies as abscissas if the values of the variable are plotted as ordinates. If graphical methods (see under heading, "Graphical Methods") "* * * convey to the eye a simplicity which does not really exist" is it not equally true that by assuming a continuous function for a small sample of discrete variables that the analysis may be needlessly complicated? Is it good practice to apply to small samples of observations, which are most common in engineering studies, the more elegant methods devised by the statistician on the basis of assumptions of an infinite number of observations of a continuous variable?

The writer does not clearly understand (see Section 7) why "* * * one is more likely to fit the wrong curve graphically than analytically." The statement appears to imply one or more assumptions which are not necessarily true: (1) That the same curve cannot be fitted by graphical as by numerical methods; (2) that the correct curve is known or even determinable; and (3) that the tests of goodness of fit can not be applied, within consistent limits of the data, to either an analytical expression or to its graphical counterpart.

The writer does not wish to imply that the analytical methods described by the author do not have their proper sphere of application, nor that after many more years of records are available they might conceivably be helpful in analyzing rainfall and run-off data. Perhaps by that time the mathematician and the engineer may have discovered even greater simplifications in both graphical and numerical methods.

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DISCUSSIONS

THE SILT PROBLEM

Discussion

By Harry G. Nickle, Jun. Am. Soc. C. E.

Harry G. Nickle,[6] Jun. Am. Soc. C. E. (by letter).[6a]—In gathering data from many sources all over the world, Mr. Stevens has shown clearly the many interrelated questions involved in the silt problem, and the need for continued research in these subjects. As the population continues to increase and it becomes necessary to conserve more fully the water resources of the United States, the problems discussed in this paper and the need for more data concerning them will become increasingly important.

Of immediate interest to those in charge of the construction of reservoirs along streams, especially in the more arid sections of the country, are the quantities of silt to be carried by streams into any proposed reservoirs and the conditions and volume of deposit of that silt in the reservoirs under various conditions of operation.

The quantity of suspended material carried by a stream can be determined with sufficient accuracy, by regular samplings, preferably taken daily, and such sampling stations should be established on all silt-laden streams which have possible reservoir sites so that as long a period of record may be obtained as possible before each reservoir is constructed. The quantity of bed-load rolled along the the bottom at any location is a more difficult problem to determine and much research remains in order to perfect a means of measuring this bed-load. The bed-load should be measured, if possible, at the same stations as the suspended silt and throughout the same period of time. The conditions and volume of silt deposits in reservoirs under different conditions of operation are known to vary widely, and much research is still necessary in this field—especially through studies and surveys of existing silt deposits—before complete information will be available. In this regard, in each reservoir investigated, a detailed study should be made of the contributing watershed, including its geology, topography, vegetal cover, and other pertinent characteristics.

Note.—The paper by J. C. Stevens, M. Am. Soc. C. E., was published in October, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[6] Asst. Irrig. Engr., U. S. Dept. of Agriculture, Board of Water Engrs., Austin, Tex.

[6a] Received by the Secretary January 8, 1935.

Mr. Stevens states that the rate of silt deposition diminishes as the reservoir fills, due to a continually lessening volume of quiet water, with the result that more silt is carried over the spillway, and also that during extreme floods earlier deposits may be picked up and carried out of the reservoir. This, of course, is true ultimately of all reservoirs. However, in any large reservoir in which the capacity above the spillway or sluice-way is large compared with the inflow, it will generally be many years before such an effect will occur to any appreciable extent. O. A. Faris, M. Am. Soc. C. E., states[7]:

"Suspended silt settles to the reservoir bottom soon after entering the slack water and, having a greater specific gravity than water, flows, in the form of liquid mud, down the slopes into depressions and along the main channel until blocked by the dam. Owing to its greater density, silt-charged water entering a reservoir partly filled with clear water does not mingle with the clear, but forces it down stream toward the dam. No suspended silt is carried through the reservoir and over the spillway until all of the clear water has been discharged."

On the other hand, in small reservoirs (those in which the capacity is relatively small compared to the inflow during floods and in which stream conditions prevail during floods rather than reservoir conditions), many different effects may occur. For instance, one flood may deposit large quantities of material and a later one may clean large volumes of this material from the reservoir; or in other cases a slope may be built on the upper side of the spillway so that heavy materials are rolled up and over it, as finer materials are carried over in suspension. The exact behavior of floods through these smaller reservoirs, the deposition and picking up of material in them, and the quantity of material carried over the spillways are subjects that could be investigated more thoroughly, because they are constantly being discussed when new small reservoirs are proposed or built.

When the water level is subject to fluctuations the rate at which silt is deposited in a reservoir also diminishes, in general. The reservoir becomes filled because a greater area of the deposited silt is exposed during the fluctuations of water surface, and, consequently, these later deposits are more dense. The increase in the density of these later deposits depends not only on the frequency of wetting and drying, but also on the nature of the particles of which the silt is composed.

Since the summer of 1924, the silt problem in Texas has been studied by the Bureau of Agricultural Engineering of the U. S. Department of Agriculture (the Division of Agricultural Engineering of the Bureau of Public Roads prior to July 1, 1934), in co-operation with the Texas Board of Water Engineers.

Tens of thousands of samples of the water have been taken from the principal streams of Texas at twenty-three regular sampling stations, and the quantity of silt has been determined at these stations over varying periods of time. Miscellaneous samples have also been taken at various other locations

[7] "The Silt Load of Texas Streams," by Orville A. Faris, M. Am. Soc. C. E., *Technical Bulletin No. 382*, U. S. Dept. of Agriculture, 1933.

throughout the State. Conditions of silt deposits in several of the reservoirs of the State have been studied in detail, and other phases of the silt problem have been investigated, including the quantity of bed-load carried by streams, the distribution of the silt throughout the cross-section of the stream, the relationship of the quantity of silt to the velocity, and the size and character of the particles composing the silt deposits, both in the reservoirs and along the stream channels. This co-operative work was under the direction of the late Robert Grier Hemphill, Assoc. M. Am. Soc. C. E., until his death in 1930; then under that of Mr. Faris until the end of 1933; and, under the writer, since April, 1934.

The results of this silt investigation to the end of the year 1930 were published in September, 1933.[7] Only a small part of the results of this intensive silt investigation in Texas have been given in Mr. Stevens' paper.

In September, 1925, at a time of low stage, a silt survey was made of Medina Reservoir, on the Medina River, about 35 miles northwest of San Antonio, Tex. The drainage area of this reservoir is 587 sq miles, the larger part of which is brush-covered grazing land, with a range in elevation of from 1 000 to 2 500 ft above sea level, and with an average rainfall of 29 in. The storage capacity at the elevation of the spillway crest was 254 000 acre-ft when the reservoir was constructed. After thirteen years of operation, in 1925, this reservoir contained 2 692 acre-ft of deposited silt, or only 1.06% of the total capacity of the reservoir when constructed. This represents a yearly average of 207 acre-ft, or 0.35 acre-ft per sq mile of water-shed per year. This deposited material had an average dry weight of 30 lb per cu ft when the survey was made in 1925, but five years later, owing to exposure to the sun and atmosphere at various times during the five years, the average dry weight of the material in place was estimated to be 63.6 lb per cu ft. In other words, the 2 692 acre-ft of deposited silt measured in 1925 had shrunk to only 1 270 acre-ft in 1930, or only 0.50% of the total capacity of the reservoir when constructed.

Measurements of the average weight of dried silt per cubic foot of material in place, taken at several reservoirs, vary from 18 to 37 lb per cu ft, when the deposited material has been under water at all times, to 85 or 100 lb per cu ft, or even higher, at locations where the deposits were subject to alternate wetting and drying, these values differing in each case not only due to different conditions of operation of the reservoirs, but also to the varying gradations and sizes of the particles of which the deposits are composed. Due to varying characteristics of large drainage areas and the varying conditions under which reservoirs are operated, it has been impossible to set a definite value for the dry weight of the deposited silt. However, after considering all factors entering into the problems, including the fact that an indeterminable quantity of vegetable matter deposits also and lasts indefinitely, a value of 70 lb per cu ft of material in place was chosen as an average ultimate figure for reservoirs in which silt deposits are subject to alternate wetting and drying.

Samples of dried silt from reservoir deposits had an average specific gravity of about 2.65, whereas samples taken from suspension and from which

TABLE 9.—SUSPENDED SILT CARRIED BY TEXAS STREAMS

Item No.	Stream	Locality	Drainage area, in square miles	Period	Number of observations in period	Quantity of water, in thousands of acre-feet	SUSPENDED SILT	
							Per thousand	Millions of tons (2,000 lb.) during period
1	Neches	Rockland	3 540	8/8-12/31/30	133	338.4	0.14	0.063
2	Double Mountain Fork, Brazos	Aspermont	7 980	6/4-12/31/24	....	10.9	21.6	0.32
				Jan.-Dec., 1925	....	130.5	28.3	5.02
				Jan.-Dec., 1926	....	314.9	19.5	8.34
				Jan.-Dec., 1927	....	64.6	18.3	1.61
				Jan. Dec., 1928	....	119.8	25.6	4.17
				Jan.-Dec., 1929	....	113.0	22.4	3.44
				Jan.-Dec., 1930	....	176.1	31.5	7.56
Total Item No. 2			....		549*	929.8	24.1	30.46
3	Salt Fork, Brazos	Aspermont	4 990	6/4-12/31/24	130*	33.2	15.5	0.70
				1/1-8/29/25	....	104.4	32.5	4.62
4	Clear Fork, Brazos	Crystal Falls	4 320	Sept.-Dec., 1925	....	105.9	2.3	0.33
				Jan.-Dec. (except June), 1926	....	139.7	1.7	0.33
				Jan.-Dec., 1927	....	125.0	2.8	0.47
				Jan.-Dec., 1928	....	338.6	3.8	1.73
Total, Item No. 4			....		966*	709.1	3.0	2.86
5	Clear Fork, Brazos	Eliasville	5 740	6/3-12/31/24	291*	98.5	3.0	0.40
				Jan.-Aug., 1925	....	122.0	3.7	0.61
6	Brazos	Seymour	14 500	6/5-12/31/24	....	78.3	18.4	1.96
				Jan.-Dec., 1925	....	398.3	23.1	12.55
				Jan.-Dec., 1926	....	605.3	16.1	13.28
				Jan.-Dec., 1927	....	100.6	10.2	1.40
				Jan.-Dec., 1928	....	225.4	20.3	6.22
				Jan.-Dec., 1929	....	231.4	16.2	5.10
				1/1-7/13/30	....	423.5	17.7	10.23
Total, Item No. 6			....		584*	2 062.8	18.1	50.74
7	Brazos	Mineral Wells	23 100	6/2-12/31/24	....	201.0	6.8	1.87
				Jan.-Dec., 1925	....	1 149.5	12.7	19.93
				Jan.-Dec., 1926	....	1 368.3	9.5	17.71
				Jan.-Dec., 1927	....	443.6	4.4	2.66
				Jan.-Dec., 1928	....	964.7	7.9	10.43
				Jan.-Dec., 1929	....	756.1	8.9	9.14
				Jan.-Dec., 1930	....	1 697.6	7.8	18.02
Total, Item No. 7			....		2 241*	6 580.8	8.9	79.76
8	Brazos	Glenrose	24 800	June-Dec., 1924	....	228.0	6.3	1.95
				Jan.-Dec., 1925	....	1 119.8	11.1	16.85
				Jan.-Dec., 1926	....	1 772.0	10.2	24.60
				Jan.-Dec., 1927	....	653.6	6.7	5.99
				July-Oct., 1928	....	452.3	5.7	3.49
				Jan.-Aug., 1929	....	516.9	8.1	5.73
Total, Item No. 8			....		858*	4 742.6	9.1	58.61
9	Brazos	Waco	28 500	June-Dec., 1924	....	293.3	4.9	1.96
				Jan.-Dec., 1925	....	1 268.8	10.9	18.86
				Jan.-Dec., 1926	....	2 307.1	9.1	28.55
				Jan.-Dec., 1927	....	1 445.9	5.5	10.83
				Jan.-Dec., 1928	....	1 375.2	8.3	15.60
				Jan.-Dec., 1929	....	1 330.3	7.6	13.70
				Jan.-Dec., 1930	....	2 460.1	7.2	24.15
Total, Item No. 9			....		1 882*	10 480.7	8.0	113.65
10	Brazos	Rosenberg	44 000	6/11-12/31/24	....	664.8	1.0	0.93
				Jan.-Dec., 1925	....	3 274.2	5.1	22.74
				Jan.-Dec., 1926	....	7 843.0	4.2	44.40
				Jan.-Dec., 1927	....	5 038.5	4.3	29.63
				Jan.-Dec., 1928	....	2 864.9	6.1	23.88
				Jan.-Dec., 1929	....	6 429.5	3.8	33.61
				Jan.-Dec., 1930	....	6 543.0	5.8	51.82
Total, Item No. 10			....		2 209*	32 657.9	4.7	207.01

* Number of observations for entire period.

TABLE 9.—(*Continued*)

Item No.	Stream	Locality	Drainage area, in square miles	Period	Number of observations in period	Quantity of water, in thousands of acre-feet	Suspended Silt: Per thousand	Suspended Silt: Millions of tons (2 000 lb.) during period
11	Little	Little River	5 250	6/8–12/31/24	....	64.3	0.3	0.024
				Jan.-Dec., 1925	....	227.0	3.5	1.07
				Jan.-Dec., 1926	....	773.7	2.1	2.21
				Jan.-Dec. 1927	....	660.4	1.8	1.59
				Jan.-Dec., 1928	....	243.8	1.6	0.53
				1/1–5/27/29	....	114.1	1.7	0.26
Total, Item No. 11	......	......	......	......	1 754*	2 083.3	2.0	5.68
12	San Gabriel	Circleville	602	6/7–12/31/24	....	22.5	0.08	0.0024
				Jan.-Dec., 1925	....	62.0	2.7	0.23
				Jan.-Dec.. 1926	....	198.0	2.6	0.70
				Jan.-Dec., 1927	....	166.5	1.6	0.37
				Jan.-Dec., 1928	....	36.5	0.3	0.015
				Jan.-Oct., 1929	....	112.8	3.3	0.51
Total, Item No. 12	......	......	......	......	1 891*	598.3	2.2	1.83
13	Colorado of Texas	San Saba	30 600	9/11–12/31/30	107	1 071.1	3.2	4.69
14	Colorado of Texas	Tow	31 100	Oct.-Dec., 1927	....	218.2	2.7	0.81
				Jan.-Dec., 1928	....	896.0	3.6	4.42
				Jan.-Dec., 1929	....	762.0	3.7	3.88
				Jan.-Dec., 1930	....	1 763.5	3.1	7.46
Total, Item No. 14	......	......	......	......	824*	3 639.7	3.3	16.57
15	Colorado of Texas	Columbus	40 800	8/3–12/31/30	126	1 671.5	3.8	8.64
16	San Antonio	Falls City	2 070	9/13–12/31/27	....	24.9	0.5	0.016
				Jan.-Dec,, 1928	....	117.3	1.8	0.29
				Jan.-Dec., 1929	....	181.2	2.1	0.51
				Jan.-Dec., 1930	....	87.2	0.5	0.056
Total, Item No. 16	......	......	......	......	1 202*	410.6	1.6	0.88
17	Nueces	Three Rivers	15 600	Oct.-Dec., 1927	....	119.1	1.1	0.18
				Jan.-Dec., 1928	....	245.1	1.5	0.51
				Jan.-Dec., 1929	....	770.4	1.2	1.29
				Jan.-Dec., 1930	....	572.4	0.9	0.73
Total, Item No. 17	......	......	......	......	1 135*	1 707.0	1.2	2.71

* Number of observations for entire period.

vegetable matter was excluded had an average specific gravity of 2.73. Mechanical analyses of various samples of suspended silt from the streams of Texas showed on the average more than 97% passing the No. 300 sieve. Of the liquid mud taken from the Medina Reservoir, 99.5% passed the No. 300 sieve; and of the silt samples taken of the deposited material at Lake Worth, only one showed greater than 0.2% retained on the No. 300 sieve.

No direct relation was found between the volume of suspended silt carried and the velocity of the stream. However, in the cases considered, capacity loads were not even approached and the quantity carried was a function of loading only.

Table 9 gives records (which are similar to, but which are not included in, Table 6 of Mr. Stevens' paper) of silt carried by the streams at the several silt-sampling stations to the end of 1930. At several of these stations, especially those in the upper reaches of the Brazos River water-shed, the flow is intermittent, sometimes many days or even several months elapsing without any flow past some of these stations.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

ELASTIC PROPERTIES OF RIVETED CONNECTIONS

Discussion

BY RALPH E. GOODWIN, ASSOC. M. AM. SOC. C. E.

RALPH E. GOODWIN,[6] ASSOC. M. AM. SOC. C. E. (by letter).[6a]—Without question, this paper is a genuine contribution to the knowledge of restrained end conditions in beams and rectangular frames and, in a timely manner, Professor Rathbun has saved students of this subject from the sin of pride and the dangers of self-complacency. Just as the profession was becoming acquainted with new means for analyzing rigid frames with some degree of facility Professor Rathbun comes forward with a bitter new "dose" in the form of further complications to problems already so complicated that few engineers other than college professors bother to solve them.

The test results are valuable and the mathematical analysis is clever and cleverly presented. The use of simple beam moments in connection with restrained end conditions in Part II of the paper simplifies the resulting expressions enormously. Further simplification results from the author's use of the symbol, L_2; but he flatters the intelligence of his readers when he assumes that the steps in his derivations and his systems of algebraic signs will be self-evident. In problems of this nature the difficulties with algebraic signs become almost unsurmountable unless the latter are explicitly defined. The tendency of experts is to grow so accustomed to their own particular methods and sign conventions that it does not occur to them that these methods and conventions may not be taken for granted by every one. Since the paper is written for specialists, some engineers who are not expert in this field will have difficulty in following the derivations and in using the resulting equations without further elucidation.

Unfortunately, the author did not make clear in his Figs. 12 to 23 the meaning of the straight diagonal dashed lines and of the straight vertical dashed lines at 0.0089 radian on some of these curves. The straight diagonal

NOTE.—The paper by J. Charles Rathbun, M. Am. Soc. C. E., was published in January, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[6] Asst. Prof., Civ. Eng., Coll. of the City of New York, New York, N. Y.

[6a] Received by the Secretary January 25, 1935.

dashed lines represent the average slopes of the "set" curves, whereas the straight vertical dashed lines indicate the rotation angle which the ends of a uniformly loaded simple beam make with the horizontal when the beam is deflected one-three-hundred-sixtieth of the span. This value of the rotation angle is thought to be the maximum limit of practical significance. In taking the "set" curves the load was removed and the point taken on these curves as the load was placed back on the specimen.

The general problem of elastic connections is similar in nature to the problem of joint rotation in the slope-deflection method. The yielding of the connection has the same effect upon the end moments of the member as would be produced by joint rotation; but in the equations the two effects must be kept distinct for two reasons: (1) Joint rotation is common to all members meeting at the joint in question, whereas yielding of the connections may not be the same for all members; and (2) yielding of any connection is itself a function of the end moment. In the conventional derivation of the slope-deflection equations for rigid connections, as presented in textbooks, the principle of superposition is utilized in order to add the fixed-end moments to the moments produced by joint rotation and joint translation. This is legitimate since the two effects are assumed to be independent of each other, the angles of joint rotation (θ_A and θ_B) not being functions of the end moments (M_A and M_B); but in the author's work the elastic rotation of a connection is itself a function of the end moment which produces it and, therefore, the ordinary fixed-end moment can not correctly be superposed upon the effects of joint rotation and translation. The author's procedure in respect to this point is correct inasmuch as he derives his slope-deflection equations for elastic connections from first principles and not by superposition. As was to be expected, the restrained-end moments for a beam with elastic connections appear in the expressions for M_A and M_B (Equations (13) and (14)),[6b] not the fixed-end moments for rigid connections. It is to be regretted as increasing the chances for error that the author has used the same symbols (M_{CA} and M_{CB}) for the restrained-end moments of a beam with elastic connections as for the fixed-end moments of a beam with rigid connections.

For moment diagrams the author wisely chooses to utilize the principle that a beam with end moments may be treated as a simple beam carrying the end moments as additional loads, so that his moment diagrams show three separate diagrams superposed, namely: (1) Moment diagram for the transverse loads on a simple beam; (2) moment diagram for end moment on the left, applied as a load on a simple beam; and (3) moment diagram for end moment on the right, applied as a load on a simple beam. This treatment appears to produce maximum simplification of the resulting equations.

In the matter of algebraic signs, always so puzzling, the author is fairly considerate. He offers much specific information in regard to this matter, but the following categorical listing of the rules used for algebraic signs in the

[6b] Corrections in paper as published in January, 1935, *Proceedings*: In Equations (11) and (12), delete "= 0"; and in Fig. 28(c) insert "M_A", thus, "$M_A\ Z_A\ (L - a)$" and "$M_A\ Z_A \left(b + \frac{c}{3}\right)$".

paper may be helpful. In the first part of the paper, as far as the "Theorem of Three Moments," the author uses a combination of two sign conventions: The moments of transverse loads follow the beam sign convention; and end couples, joint rotation, joint translation, and yielding of connections follow the slope-deflection sign convention, counter-clockwise rotation being taken as positive. Although it is immaterial for the cases discussed in this paper the writer would like, on general principles, to amend the statement (following Equation (5)) that "the moment is positive when it tends to turn the member in a counter-clockwise direction." A counter-clockwise end moment should always be taken as positive whether acting on the member or on the adjacent joint, but it should be indicated whether the symbols, M_A, M_B, etc., refer to the moment couples acting on the member or to the ones acting on the joint. In the paper the symbols, M_A, M_B, etc., refer to the moment couples acting on the member, so that a positive value of M_A indicates a counter-clockwise couple acting on the member with a corresponding clockwise couple ($-M_A$) acting on the adjacent joint. In plotting moment diagrams when using the direction-of-rotation sign convention the several component diagrams must all be plotted consistently; that is, all must refer to the same face of a section through the member. As regards algebraic signs the author's moment diagrams show the moment couples acting on the left-hand face of a transverse cut through the beam. On this face of the section, the algebraic signs of the moment couples will be the same when determined by the direction-of-rotation sign convention as when determined by the beam sign convention. The foregoing remarks explain the author's statement (under the heading, "Derivation of Formulas") that: "It is to be noted that although M_A is positive, it produces a negative moment on the moment curve." Some may consider it objectionable to combine two different systems of algebraic signs as has been done in the paper, but no difficulty will be experienced in altering the equations by those who desire to make a change in this particular.

In the derivation of the theorem of three moments with elastic connections the beam sign convention is used as more appropriate to this case. In his work the author simplifies his mathematics by treating Joints A and C as hinged and acted upon by the couples, M_A and M_C, considered as loads. He allows for the effect of these couples by treating them as loads on a simple beam and including their effect with that of the transverse loads. In the case of a hinged connection at A, L_{2A} becomes infinite and Equation (14) becomes indeterminate in the form in which it is printed. Equation (28) is obtained by first dividing both numerator and denominator of Equation (14) by L_{2A} and then setting L_{2A} equal to infinity. The equation for L_{BA} M_B may be obtained in a different manner by eliminating θ_A from Equations (13) and (14). Similar procedure is followed in obtaining Equations (29) and (30). The assumption is made in the derivation that the differences in elevation between Supports B and A and Supports B and C are the same and that each is equal to Δ. This is equivalent to rotating the pair of spans until Supports A and C are at the same elevation; or, which is the same thing, assuming as a reference axis the line joining Supports A and C. The result

is valid when all three supports are at different elevations, and it may be shown that,

$$\Delta\left(\frac{1}{L_a}+\frac{1}{L_b}\right)=\left(\frac{\Delta_a}{L_a}+\frac{\Delta_b}{L_b}\right)\ldots\ldots\ldots\ldots\ldots\ldots(42)$$

Perhaps the mathematical part of the paper would appeal to a wider public if the author had been more liberal with explanations, but the subject-matter is of the highest interest, and the purpose of the present discussion is mainly to endorse the paper and to clarify it for the benefit of those who are not expert in this field. Any suggestions made herein are of trivial weight in comparison with the importance of the original work.

In Equations (26), (27),[6e] and (33), as well as in their derivation, the signs of the moments are assumed positive when they produce compression in the top fibers of the beam rather than by the convention used in "Slope-Deflection Method." This is done to be consistent with current practice in the use of the theorem of three moments. In Equation (29), M_A is negative as are the first and second quantities in the right-hand member of Equation (32). In the sentence preceding Equation (30), change "Equation (27)" to "Equation (29)" and "$-M_{BA}\ Ax_1$" to "$M_{BC}\ Ax_1$"; in the line preceding Equation (40), change "$\div \frac{1}{2}$" to "$\div \frac{1}{Z}$"; and, in Fig. 34, change "*t*" to "*i*" and "*w*" to "*j*".

[6e] Correction in paper as published in January, 1935, *Proceedings*: Page 33, line 9, change "Fig. 32" to "Fig. 27".

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

SOCIETY AFFAIRS

ANNUAL REPORT OF THE BOARD OF DIRECTION FOR THE YEAR ENDING DECEMBER 31, 1934

In compliance with the Constitution, the Board of Direction presents its Report for the year ending December 31, 1934.

THE EIGHTY-SECOND YEAR

For four full years, from 1931 to 1934, inclusive, civil engineers as a group have been victims of the depression. The year 1930 was the peak of all-time employment of civil engineers and the peak of all-time civil engineering salaries. The year 1933 found approximately 50% of the civil engineers of the country without employment and the income of the profession shrunken to an even greater degree. In October, 1933, the Civil Works Administration afforded re-employment, but at relief wages. Later, the Federal program of Public Works, and the Federal Emergency Relief Administration in their various ramifications, gave employment to many more. The close of 1934 found a large percentage of the civil engineers of the country at work, at something, although with greatly curtailed income and to an unfortunate extent upon work in which their capabilities have not been utilized to the full. It has been demonstrated, as a type characteristic of the Civil Engineer, that he prefers to be busily engaged at something even if he is paid little for it, or even if it is not to his liking.

Re-Employment

The activities of the Society during the past four years, and even more, have been addressed to such an extent to efforts to support salaries and the business conditions affecting civil engineers and to induce re-employment of engineers that this report might be devoted entirely to this feature; even then it would not afford more than an outline of what has been attempted and what has been accomplished.

Major programs, like those of establishing the Federal Public Works, and the problem of an Engineering Division of the Code of Fair Practice within the Construction Industry, were reported on last year. This year attention may be given to the diversity of efforts put forth by the Society's Board of Direc-

tion, its officers, and its Local Sections to effect re-employment. A complete list of topics, even without details, would be long.

The surveys made by the Society's Committee on Salaries merit particular attention. First, the survey by which was determined, with a high degree of accuracy and authority, the salaries paid throughout the country in good times —1930. Next, a survey, made in 1933, of the unemployment that had occurred and that was to be expected among civil engineers. Third, a record of what had been observed to be the current salaries, and below which it was unfair to subject civil engineers as a segregated group of employees, and, finally, the report on Prevailing Salaries of Civil Engineers. This report was produced to meet a need felt by Federal officials and was made in order that there might be support for them in their rulings to the effect that salaries paid were to conform to those prevailing in the different localities. The efforts of Society officers to obtain these rulings and to see that injustices were rectified should also be recorded. The Society is now assisting in a comprehensive survey of the entire Engineering Profession looking to the determination of many facts in behalf of the engineer.

An expansion of the Federal mapping program has been persistently urged since 1931, the first approach being to the President of the United States. Under the Civil Works Administration a program of local control surveys under the direction of the United States Coast and Geodetic Survey gave employment to 14 000 men, 10 000 of whom were engineers. During the Congress of 1933, a representative was placed in Washington, at Society expense, to assist in securing a permanent expansion of this work and, during the Congress of 1934, an appropriation of $22 000 000 was sponsored by the Society's officers, who frequently visited Washington for this purpose. The Society's officers are now assisting in the proposal that the mapping of the country shall proceed at double its normal speed.

Conditions of employment in Russia have been studied and made known to the members; Lien laws, uniform throughout the United States, which would protect the engineer, were supported through American Engineering Council; legislation affecting the construction industry, including the part held by the engineer and the architect, was supported through the Construction League of the United States; civil engineers found employment in the reforestation program, with the Civilian Conservation Corps, the National Park Service, the administrative staffs of the Reconstruction Finance Corporation, the Public Works Administration, the Civil Works Administration, and the Federal Emergency Relief Administration through the efforts of committees financed by the Society, Society officials, and groups of selected Society members.

Protest has been made by the Board of Direction, and actively pursued by its officers, against the competition by the Federal Government with engineers in private practice. Also, protest was made against the requirement of party "clearance" for appointments. Protests have been made against the performance of regular engineering work at relief wages; and against the substitution of workers at relief wages for Civil Service engineer employees discharged or demoted upon one pretext or another; against the practice within the Home

Owners' Loan Corporation of not paying for lot surveys, made by engineers, on which loans have been declined; against the making of changes in engineers' designs without their approval, by the Supervising Architect of the Treasury; and against deductions from engineers' fees for the cost of such changes.

The Engineering Societies Employment Service has been maintained jointly with other Engineering Societies since 1918, the Society contributing to its support in the institution of its three offices, and when non-self-supporting, in bad times, in excess of $42 000. Local Relief Committees formed by the Local Sections have surveyed the economic conditions of engineers in their respective territories and have placed several thousand engineers in remunerative work of one kind or another. At places remote from organized Local Sections, a national relief fund has been made available to engineers in straitened circumstances.

Throughout the past year especially, letters were sent at frequent intervals to the Presidents and Secretaries of each of the Society's fifty-seven Local Sections advising them of the Federal procedures initiated or rulings made which might be of assistance in the re-employment of engineers or of assistance to engineers in private practice. Information thus spread abroad gave opportunity for Local Section committees or selected individuals to acquaint promptly both members and non-members all over the country with information of which they could avail themselves.

Since 1925 the Society's Committee on Registration of Engineers has made reports and provided leadership, to other groups interested in the registration of engineers, in the development of a Model Registration Law. The Board of Direction has given its endorsement to the registration of engineers and urged upon its Local Sections active participation where registration legislation is proposed in the several States.

Manuals of Practice, produced by the Society's Committee on Fees, have acquainted engineers in private practice with the situations that are likely to confront them and have established rates of fees by which they may be protected from loss or their employees from unfairly low salaries. The Society's Committee on Public Education, by prepared articles, by releasing Society papers to the press, and by radio talks has endeavored to convey to the public a favorable impression of the engineer. Through co-operation in the Engineers' Council for Professional Development the Society has assisted in a profession-wide program of vocational guidance for young men, of accrediting engineering schools, of professional training, and of professional recognition.

Thus, without diversion of its attention from the development of the highest possible technical literature, and without lowering its requirements for membership, the funds contributed by its members have made possible, through committees and individual effort within the past ten or twelve years, a wide program devoted to the improvement of the economic condition of the civil engineers of the country. In the past three or four years, and particularly in the year just closed, these efforts have been exerted promptly and energetically as exigencies or opportunities arose. Their success is attested by the extent to which members of the Society and non-members alike have been re-employed

in this time of depression, even if not to their full satisfaction nor in positions which permit full expression of their capabilities.

Membership

That membership in the Society is highly prized is evident from many sources. The number elected as Juniors was 40% in excess of last year, and, of the older men in the Associate Member and Member grades, there was a 29% increase. As has been the custom for the past three years, again this year the Board of Direction established a "suspended dues list" available to those members who had previously loyally supported the Society, but who in these times found it impracticable to pay current dues. This plan extended to all grades of membership for it has become a recognized principle that the Society's real value is its "assets in men—not money". Dues thus waived, amounting to $57 773, were written off the books this year. A total of approximately $110 000 has been written off, during the years of the depression, in order that membership opportunities may be maintained for those who have had long connection with the Society. The net membership is 291 lower than last year, due largely to the dropping of 618 members who were in arrears and had paid dues for but one, two or only a very few years. The Board believed it unfair to permit these to continue as members with full privileges while others had continued their payments even though that may have been a real burden. In the latter part of the year reinstatement provisions were devised and made known to all whose membership had ceased during the past five years, a great many of them by resignation, being fully paid up, but not in position to continue financially. The adopted reinstatement provisions will relieve them of the payment of a second entrance fee and of dues for the period during which they received no benefits from the Society. Reinstatements thus made will not be reflected in the membership total until 1935. The total membership on December 31 was 14 910.

Finances

At the beginning of the year the Budget was set up to effect expenditures $21 642 in excess of anticipated receipts. Economies resulted from continued reduction in the size of *Proceedings,* somewhat smaller issues of *Civil Engineering,* decrease in the number of meetings of the Board of Direction and of the Society, continued reduction in salaries of the Staff, etc. Adjustment of the Budget, made from time to time during the year, especially to the furtherance of new Committee work, increased the expenditures, but none the less the outcome for the year was not a deficit. Again it was the appreciation of the value of the Society to them by the members and their loyalty to it which reversed the circumstances, at least in part. Re-employment of members increased the anticipated income from dues by $22 500 and the increase in the anticipated advertising in *Civil Engineering* contributed a large part of the remainder. Both items indicate improvement in general business conditions. Final payment was made on the mortgage, and the Society is now without any unpaid financial obligation of any kind. It should go forward rapidly in greatly increased service to its members and to the profession.

MEETINGS OF THE BOARD OF DIRECTION

There have been five meetings of the Board of Direction during 1934:

January 15–16, New York, N. Y.
January 18, New York, N. Y.
May 11, New York, N. Y.
July 9–10, Vancouver, B. C., Canada.
October 1–2, Chicago, Ill.

There have been four meetings of the Executive Committee:

March 23, Washington, D. C.
July 9, Vancouver, B. C., Canada.
October 1, Chicago, Ill.
December 21, New York, N. Y.

MEMBERSHIP

The changes in membership are shown in the following table:

	Jan. 1, 1934			Jan. 1, 1935			Losses				Additions			Totals		
	Resident	Non-Resident	Total	Resident	Non-Resident	Total	Transfer	Resignation	Dropped	Died	Transfer	Election	Reinstatement	Loss	Gain	Decrease
Honorary Members...	5	13	18	6	12	18	0	0	0	3	*3	0	0	3	3	0
Members............	964	4 789	5 753	945	4 742	5 687	3	33	48	118	†84	43	9	202	136	66
Associate Members....	1 050	5 220	6 270	966	5 105	6 071	84	62	272	39	‡97	144	17	457	258	199
Juniors............	547	2 499	3 046	517	2 515	3 032	97	64	§387	7	0	528	13	555	541	14
Affiliates..........	34	75	109	32	66	98	0	3	8	3	0	2	1	14	3	11
Fellows............	3	2	5	2	2	4	0	0	0	1	0	0	0	1	0	1
Total...........	2 603	12 598	15 201	2 468	12 442	14 910	184	162	715	171	184	717	40	1 232	941	291

* 3 Members.
† 84 Associate members.
‡ 97 Juniors.
§ 97 Juniors dropped on account of age limit.

New Members and Net Increase

The following table shows the new members and the net increase during the past ten years. The diagram on page 6 gives membership statistics for the same period:

	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934
New Members*..	795	1 072	1 139	1 244	1 066	1 139	1 055	753	534	757
Net Increase....	111	721	755	820	508	574	531	57	46§	291§

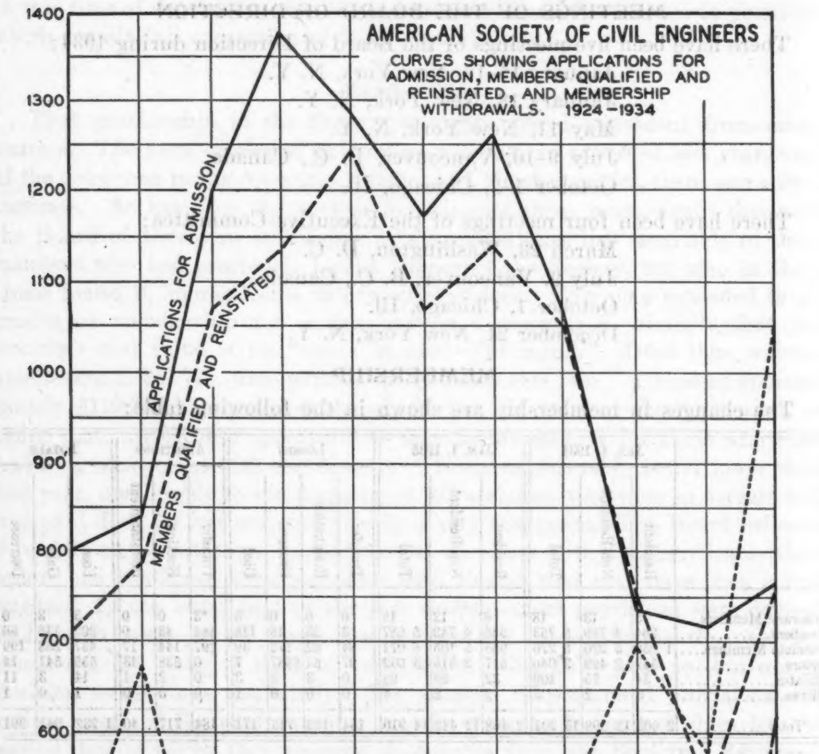
* Includes reinstatements.
§ Decrease.

Applications for Membership

The total number of applications for membership was 990, of which 768 were for admission, including 2 for re-admission, and 222 for transfer.

The number of applications received during the past ten years follows:

	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934
For admission.....	848	1 194	1 374	1 284	1 172	1 260	1 072	736	715	768
For transfer.......	256	292	304	274	271	338	224	192	172	222
Total.........	1 104	1 486	1 678	1 558	1 443	1 598	1 296	928	887	990



CURVES SHOWING NEW MEMBERS AND NET INCREASE IN MEMBERSHIP 1924–1934.

DEATHS

The losses by death during the year number 171, and are as follows:

Past-Presidents (1):
Carl Ewald Grunsky

Honorary Members (3):
Baron Koi Furuichi
Milo Smith Ketchum
Palmer Chamberlaine Ricketts

Members (117):
Octavio Augusto Acevedo
Frederick Whitney Adgate
James Pierson Allen
Frederick James Amweg
Kort Berle
Howard Colburn Blake
Jared Sperry Bogardus
Alfred William Bowie
Jules Breuchaud
Wilbur Gayle Brown
George Hamilton Browne
George Bryan, Jr.
Henry Robinson Buck
Edward Morris Burd
William Hubert Burr
John Soule Butler
Walter John Cahill
Alexander Cahn
Henry Hall Carter
James Russell Chapman
William Arnold Christian
Fred Bush Church
Charles Samuel Churchill
David Clark
George Hallett Clark
Lorenzo Dana Cornish
William Howard Courtenay
Clyde Maxwell Cram
Benjamin John Curtis
William Clarence Davidson
Robert Benjamin Davis
Henry Delaplaine
John Thomas Donaghey
Edward John Duffies
Arthur Chester Eaton
Frederick Eaton
George Alexander Miller Elliott
Eric Gustaf Ericson
Louis Biaut Eyquem
Walter Linder Foster
Felix Freyhold
George Warren Fuller
William Gore
Samuel Martin Green
Victor Hugo Greisser
William Albert Grover
John Wendell Hall
William Hammond Hall
Van Alen Harris
Frederick Nathaniel Hatch
Robert Graham Hengst
Fred Forbes Henshaw
Charles Edward Hewitt
John Edward Hill
Elmer Guy Hooper
George Edward Howe
William Henry Hull
George Arthur Johnson
James Moreland Johnson
Percy Francis Jones
Earl Wallace Kelly
Maurice Edwin Kernot
Wynkoop Kiersted
Victor Hugo Kriegshaber
Oliver Howard Lang
William Campbell Langfitt
William States Lee
Frederick William Lovell
Frederic William Lyon
Arthur Hicks McCarrell
Richard Justin McCarty
Luis Matamoros
Richard Levi Miller
Edward Charles Murphy
Lars Netland
Maury Nicholson
Harry Alonzo Noble
Emil Louis Nuebling
Francis Joseph O'Hara
John Francis O'Rourke
Michael Maurice O'Shaughnessy
Sir Frederick Palmer
Walter Camp Parmley
William Merit Penniman
Charles Penrose Perkins
Thomas Pettersen
Albert Waring Pierson
Henry Sewall Prichard
Louis E. Ritter
Edwin John Rosencrans
Hiram Newton Savage
Otto Stephen Schlich
Hiram Abif Schofield
Philip Kingsland Schuyler
Paul Albert Nicolas Seurot
George Harry Thornton Shaw
Eugene Hicks Shipman
Joseph Mansfield Slater
Kenneth Irving Small
Burton Smith
Theodore Speiden
Frank Julian Sprague
Gustave Arnold Stierlin
John Francis Sullivan
Milo Cornelius Taylor
John Arnold Ubsdell
Aaron Howell Van Cleve
Edmund French Van Hoesen
Alexander Wilson Vars
Edwin Derickson Vincent
John Douglas Waldrop
Joseph Houston Wasson
Robert Malcolm Watson
Edwin John Weigand
Thomas William White
Herbert Alva Wilson
Clarence George Wrentmore

Associate Members (39):
Joseph Anthony Baker
Harold Ward Barker
Earl Clarence Brown
James Hopkins Clark
Joseph Augustine Aloysius Connelly
Herbert Allen Dunlap
Charles Wesley Erisman
Frederic Morris Faude
Frank Lewis Gartrell
Elbert Allan Gibbs
John Wesley Goodridge
Elmer Clark Goodwin
John Francis Grady
George Waldo Hillman
Walter Jacob King
Warren Raymond King
Isaac Henry Kirby
Harry Kornfeld
Andrew Peter Ludberg
David Livingstone MacBeath
James Brownson McClain
Arthur Sawyer Mahony
Thomas Vincent Moore
William Arnold Newman
Courtland Nixon
Herbert Paterson
George Abel Pierce
Ira Taylor Redfern
Herbert Allan Rice
Albert Rosenthal
George Benjamin Rule
Herbert Davis Schutt
Edson Oliver Sessions
Albert Orange Smith
Gilbert Cobb Staehle
Gustaf Alexander Stenström
Francis Alexander Stewart
Burtis Paul Thomas
John Summie Whitener

Juniors (7):
Joseph Ingham Francis
Thomas Jefferson Hayden, Jr.
Frank Firth Hord
John Alexander Jameson, Jr.
Oliver Bertrand Johnson
Paul Van Rush
Herman Greig Veeder, Jr.

Affiliates (3):
Frank Hasbrouck Earle
Charles Guy Eldredge
Edward Kimball Fenno

Fellows (1):
Charles R. Flint

ENGINEERING SOCIETIES LIBRARY

The statistics which follow give comparative figures for 1933 and 1934 of the Engineering Societies Library:

	1933		1934	
Additions:				
Volumes (by gift).................	1 789		1 825	
" (by purchase)	1 124	2 913	996	2 821
Pamphlets (by gift)	3 339		3 371	
" (by purchase)	272	3 611	214	3 585
Maps (by gift).................	141		140	
" (by purchase)	8	149	9	149
Searches		26		21
Total additions.................		6 699		6 576
Permanent collection		143 660		145 620
Expenditures for books, periodicals, binding, supplies, and salaries (approximate).................		$40 077		$40 002
The Library was used by.................		42 915		40 789
Including personal visits by.................		33 258		29 928
Volumes catalogued		3 360		2 821
Cards added to catalog.................		17 237		18 327
Total catalog cards, arranged under subject........		465 016		479 435
Searches made		49		68
Translations made		97		109
Photoprints made		16 889		18 493
Number of persons securing photographs..........		2 215		2 366
Receipts for service.................		$8 904		$7 766
Members borrowing books.................		111		127

EMPLOYMENT SERVICE

The Employment Service has offices in New York, N. Y., Chicago, Ill., and San Francisco, Calif.

The number of men placed during 1934 has averaged about 139 per month. The following table shows the registrations and placements in the three offices:

Month	Men Registered				Men Placed			
	New York	Chicago	San Francisco	Total	New York	Chicago	San Francisco	Total
January.....	67	1 002	44	1 113	60	644	17	721
February....	66	74	37	177	38	80	9	127
March.......	81	33	69	183	48	23	10	81
April........	93	49	48	190	54	15	17	86
May.........	86	71	55	212	46	19	16	81
June........	145	76	66	287	50	9	8	67
July.........	129	47	46	222	51	18	13	82
August......	85	47	45	177	61	22	17	100
September...	99	28	34	161	51	12	13	76
October.....	113	41	58	212	49	40	16	105
November...	90	38	37	165	41	23	10	74
December...	72	41	31	144	42	12	15	69
Total...	1 126	1 547	570	3 243	591	917	161	1 669

PUBLICATIONS

The publications of the Society for 1934, include one volume of *Transactions* (Volume 99), ten numbers of *Proceedings,* twelve numbers of *Civil Engineering,* three Manuals, an Index to *Transactions,* and a Year Book.

Transactions.—Volume 99 is the regular yearly issue of *Transactions* and includes the papers and discussions published in *Proceedings* from August, 1932, through October, 1933. The volume also contains the Annual Address of President Harrison P. Eddy, the Final Report of the Special Committee on Irrigation Hydraulics, and Memoirs of Deceased Members. Volume 99 of *Transactions* was issued as Part 2 of October, 1934, *Proceedings.*

Proceedings.—During the year the Final Report of the Special Committee on Irrigation Hydraulics was published in the March *Proceedings.* There were also published in *Proceedings,* 32 papers and 1 Symposium, together with the discussions thereon, as well as a number of discussions of papers published in 1932 and 1933. Subject and Author Indexes for the year were included in the December number.

Members and others who took part in the preparation and discussions of these papers, symposium, report, and the discussions thereon, totaled 293.

Civil Engineering.—Twelve numbers of *Civil Engineering* were issued for the year; one of which, the March number, contained the abstracts of papers and reports of committees delivered before the Annual Meeting of the Society in January, 1934; and another, the September number, contained the abstracts from the Vancouver Convention held in July, 1934. The remaining ten were regular numbers. Including the abstracts and committee reports 115 articles in all were published during the year. In addition to these, 29 brief articles appeared under the heading, "Engineers' Notebook," and 70 discussions of articles and comments from readers were published in the department headed "Our Readers Say." Concerning "Society Affairs," 116 separate items were printed; two complete programs of the meetings of the Society held during the year; and 57 "Items of Interest" not specifically relating to activities of the Society. Also were included reports of 152 Local Section meetings, and 111 reports from Student Chapters on their activities. Previews of 34 papers to appear in *Proceedings* were published; together with 135 reviews of books recently donated to the Library; and more than 1 300 brief items indexing selected articles as they appeared month by month in the technical periodical literature. During the year 323 changes of addresses or employment of members were recorded; notices concerning the availability of 363 engineers for employment appeared; and in the December number a complete index of the contents of the twelve numbers in Volume 4 was published.

Memoirs.—The publication of memoirs of deceased members in *Proceedings* was discontinued in October, 1930. A pamphlet form of memoir was adopted at that time, with final publication in *Transactions.* Approximately 500 memoirs have been issued since that time, many of which have been included in *Transactions* Vol. 95 (1931), Vol. 96 (1932), Vol. 98 (1933), and Vol. 99 (1934).

Stock of Publications.—The stock of the various publications of the Society kept on hand for the convenience of members and others now amounts to 205 667 copies, the cost of which to the Society for paper and press work only has been \$36 717.46 which allowing for depreciation and obsolescence is carried on the Society's books at a valuation of \$14 717.46.

Cost of Publications.—The table (see page 11) shows the cost per page and illustrations in *Proceedings* and *Transactions* for the past sixteen years (since and including 1919) and in *Civil Engineering* for the past five years.

The various topics developed in *Transactions, Proceedings,* and *Civil Engineering* during the year and the number of pages devoted to each are, as follows:

Subject	*Transactions,* pages	*Proceedings,* pages	*Civil Engineering,* pages
Airports			5
Bulkheads			10
City and Regional Planning	15		
Contracts	16		3
Corrosion			5
Dams	311	180	28
Drainage and Irrigation	85	140	12
Earth Pressure			8
Earth Work		14	
Engineering History	95		
Engineering Instruments			5
Engineering Profession	8		11
Erosion			4
Etymology			4
Excavation			5
Floods			7
Forestation	111		
Foundations		74	7
Government Ownership			9
Graphical Methods			7
Highway Engineering	31	59	38
Hydrology, Hydraulics	175	187	34
Machinery			7
Materials of Construction			20
Materials of Engineering	29		
Memoirs	220		
Meteorology		164	
Parks and Parkways			8
Pipe Lines			5
Power Plants			4
Railways	60		23
Real Estate Surveys			2
Refuse Disposal			5
Sewage Disposal	48	37	8
Soils	52	56	4
Structural Engineering	284	470	78
Surveying	41	14	15
Transportation	29	4	15
Tunnels			12
Water Power			13
Water Supply		22	33
Waterways		27	9
Water-Works		93	
Letters to the Editor			31
Society Affairs		23	90
Items of Interest			28
Local Sections			13
Student Chapters			10
Current Periodical Literature			36
Recent Books			6
Men and Positions Available			15
Changes in Membership Grades			14
News of Engineers			10
	1 610	1 564	716

TABLE SHOWING NUMBER AND COST OF PAGES AND COST OF ILLUSTRATIONS FOR *Transactions, Proceedings,* AND *Civil Engineering.*

Year	*TRANSACTIONS*				*PROCEEDINGS, PART I**				*PROCEEDINGS (PART I)** and *TRANSACTIONS*			ILLUSTRATIONS		
	Issues	Edition	Pages: Per volume	Pages: Total	Issues	Edition	Pages: Per volume	Pages: Total	Total pages	Total cost	Cost per thousand pages	Cost	Percentage of total cost	Cost per thousand pages
1919	1	9 000	1 775	15 980 000	8	9 100	2 096	19 075 000	35 055 000	$32 082.69	$0.91	$1 128.53	3.5	$0.032
1920	..	......	......	....	10	10 142	2 014	20 440 000	20 440 000	23 446 34	1.15	2 552.27	10.9	0.125
1921	2	10 000 10 500	2 479 993	35 212 000	10	10 680	1 834	19 450 000	54 662 000	66 298.39	1.21	2 034.72	3.1	0.037
1922	1	10 900	1 826	19 900 000	10	11 100	2 740	30 400 000	50 300 000	56 200.00	1.12	3 700.00	6.6	0.073
1923	1	11 200	1 808	20 250 000	10	11 500	3 210	36 915 000	57 165 000	60 612.83	1.06	4 809.88	7.9	0.084
1924	1	11 500	1 515	17 440 000	10	11 750	2 612	30 700 000	48 140 000	47 573.72	0.99	4 579.04	9.6	0.095
1925	1	11 400	1 538	17 533 000	10	11 800	2 910	34 338 000	51 871 000	47 409 07	0.91	6 359.19	13.4	0.122
1926	..	12 100	1 786	21 610 000	10	12 200	3 046	37 161 000	58 771 000	53 473.54	0.91	5 080.91	9.5	0.087
1927	2	12 900 13 200	1 234 1 229	15 919 000 16 223 000	10	13 050	3 720	48 546 000	80 688 000	65 972.76	0.80	9 279.49	14.0	0.126
1928	1	14 000	1 860	26 040 000	10	14 200	4 016	57 027 000	83 067 000	61 587.98	0.74	7 249.00	12.4	0.087
1929	1	14 500	2 030	29 435 000	10	14 540	3 566	51 850 000	81 285 000	58 388.39	0.72	4 011.95	6.9	0.049
1930	1	15 100	1 794	27 090 000	10	15 170	2 936	44 540 000	71 630 000	52 881.95	0.74	4 189.56	7.9	0.058
1931	1	15 500	1 720	26 660 000	10	15 820	1 968	31 133 000	57 793 000	43 579.42	0.75	3 680.95	8.5	0.064
1932	1	15 800	1 688	26 670 000	10	15 700	2 160	34 010 000	60 680 000	37 348.87	0.62	4 830.47	12.9	0.080
1933	2	12 000 12 700	448 1 752	5 378 000 22 303 000	10	13 520	1 784	24 075 000	51 756 000	32 402.35	0.63	3 653.66	11.3	0.071
1934	1	13 500	1 664	22 522 000	10	13 500	1 648	22 256 000	44 778 000	28 837.59	0.64	3 154.06	10.9	0.070
					CIVIL ENGINEERING									
1930	..	......	.....	....	3	16 230	248	4 026 000	4 026 000	$8 969.66	$2.23	$995.06	10.1	$0.247
1931	..	......	.....	....	12	16 275	1 264	20 572 000	20 572 000	41 012.75	1.99	6 271.40	15.3	0.305
1932	..	......	.....	....	12	16 020	1 048	16 786 000	16 786 000	33 057.95	1.97	5 112.68	15.4	0.305
1933	..	......	.....	....	12	13 500	912	12 316 000	12 316 000	23 678.54	1.92	3 762.72	15.9	0.305
1934	..	......	.....	....	12	13 580	954	12 956 000	12 956 000	27 423.67†	2.11†	4 426.83	16.3	0.341

* Includes Part III, May 1928, 288 pp.; Part 2, May 1932, 112 pp.
† Includes stock and supplies for following year.

Summary of Publications for 1934

	Issues	Average edition	Total pages	Cuts
Proceedings (monthly numbers)	10	13 510	1 648	489
Civil Engineering (monthly numbers)	12	13 583	906	849
Transactions Vol. 99	1	13 500	1 658	437
Manual No. 8	1	13 500	24	......
Manual No. 9	1	13 800	20	13
Manual No. 10	1	16 000	126	23
Year Book	1	15 800	460	3
Index to *Transactions,* Vol. 84–99	1	14 200	188	......
Total	28	......	5 030	1 814

The gross cost of publications, as determined by the bills actually paid during the year, and inclusive of salaries, has been:

Technical Publications	$108 528.04
General Publications	5 798.33
Total	$114 326.37

READING ROOM OF THE SOCIETY

The attendance at the Reading Room during the year was 2 148.

Two hundred and forty-four periodicals are regularly received. Included in this number are many foreign periodicals, also a number of literary magazines and several daily newspapers.

MEETINGS

Six meetings of 6 sessions were held during the year, as follows: At the Annual Meeting, at New York, N. Y., 2 (2 sessions); at the Annual Convention, at Vancouver, B. C., Canada (held jointly with the Engineering Institute of Canada), 2 (2 sessions); and 2 regular meetings held in Engineering Societies Building, New York, N. Y.

At these meetings there were presented one Paper, one Symposium, four Reports of Committees of the Society, and five Addresses.

The total attendance at the meetings of the Society during the year was approximately 2 025. The registered attendance at the Annual Meeting was 1 550, and at the Annual Convention, 475.

The dates of the meetings of the Society during the year, together with the titles of the Paper, Symposium, Reports, Addresses, etc., presented thereat, are as follows:

January 17, 1934 (Two Sessions): Reports of Committees on Concrete and Reinforced Concrete Arches; Earths and Foundations; Irrigation Hydraulics[1]; and Meteorological Data; and a Paper on "The Equitable Theory of Governmental Ownership and Operation", by Frederick H. McDonald,[2] M. Am. Soc. C. E.

March 14, 1934 (One Session): Business Meeting of the Society.

[1] *Transactions,* Am. Soc. C. E., Vol. 99 (1934), p. 1375.
[2] *Civil Engineering,* March, 1934, p. 119.

July 11, 1934 (Two Sessions): "Trends in Engineering as a Profession in the United States of America", Address by Harrison P. Eddy, President, Am. Soc. C. E.[3]; "Through British Columbia Waters to Alaska", by Commander B. L. Johnson, R. N. R.; and Symposium on "The Development of the Columbia River Basin".[4]

October 10, 1934 (One Session): Business Meeting of the Society.

MEDALS, PRIZES, AND AWARDS

The award of Medals and Prizes for the year ending July, 1934, was as follows:

The Norman Medal to Leon S. Moisseiff, M. Am. Soc. C. E., for his paper entitled "George Washington Bridge—Design of the Towers."

The J. James R. Croes Medal to H. M. Westergaard, M. Am. Soc. C. E., for his paper entitled "Water Pressures on Dams During Earthquakes."

The Thomas Fitch Rowland Prize to Miles I. Killmer, M. Am. Soc. C. E., for his paper entitled "Fulton Street, East River Tunnels, NewYork, N. Y."

The James Laurie Prize to E. W. Bowden, M. Am. Soc. C. E., and H. R. Seely, Assoc. M. Am. Soc. C. E., for their paper entitled "George Washington Bridge—Construction of the Steel Superstructure."

The Arthur M. Wellington Prize to J. C. Evans, Esq., for his paper entitled "George Washington Bridge—Approaches and Highway Connections."

The Collingwood Prize for Juniors to G. H. Hickox and G. O. Wessenauer, Juniors Am. Soc. C. E., for their paper entitled "Application of Duration Curves to Hydro-Electric Studies."

LOCAL SECTIONS

There are at present 57 Local Sections, the South Carolina Section approved by the Board of Direction on October 3, 1934, having been added during the year. The name of the Chattanooga Section was changed to Tennessee Valley Section.

TECHNICAL DIVISIONS

All but two of the Technical Divisions of the Society held sessions during the year, either at the Annual Meeting in New York in January, or at the Annual Convention in Vancouver, B. C., Canada, in July. Of these meetings two were double sessions, of which two were held jointly with similar Divisions of this, or other Societies, at the Annual Convention.

The meetings of the Divisions were marked by good attendance and interest and by an excellent group of technical papers.

City Planning Division

January 18, 1934, "Benefits and Advantages of National Planning with Reference to Emergency Conditions," by Carey H. Brown, M. Am. Soc. C. E.; "How National Planning Can Help In: (*a*) "The Development of the Major Highway System," by E. W. James, M. Am. Soc. C. E.; (*b*) "Development and Coordination of Rail, Highway, and Waterway Systems," by C. O. Sherrill, M. Am. Soc. C. E.; and (*c*) "Conservation of Natural Resources," by Arthur E. Morgan, M. Am. Soc. C. E.

[3] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1383.

[4] *Civil Engineering*, September, 1934, p. 443 *et seq.*

Construction Division

January 18, 1934, "Functions of the Construction Engineer (A Symposium): (*a*) "The Engineering Viewpoint," by Herbert S. Crocker, Past-President, Am. Soc. C. E.; (*b*) "The Field Construction Viewpoint," by A. P. Greensfelder, M. Am. Soc. C. E.; and "Constructive Engineering," by Hugh Miller, M. Am. Soc. C. E.

Engineering-Economics and Finance Division

This Division sponsored the general session at the time of the Annual Meeting, on Wednesday afternoon, January 17, 1934, consisting of a Symposium on The Equitable Theory of Governmental Ownership and Operation, by Frederick H. McDonald, M. Am. Soc. C. E., James P. Gifford, Esq., and Charles Keller, M. Am. Soc. C. E.

Highway Division

January 18, 1934, "Traffic Influence of the West Side Elevated Highway, New York, N. Y.," by Benjamin Schwerin, Assoc. M. Am. Soc. C. E.; and "New Jersey Highway Approach to Holland Tunnel," by W. G. Sloan, M. Am. Soc. C. E.

July, 12, 1934 (Joint Session with Construction Division), "Alaska-United States Highway," by Malcolm Elliott, M. Am. Soc. C. E.; "Inter-American Highway," by E. W. James, M. Am. Soc. C. E.; "The Fort Peck Project," by Theodore Wyman, Jr., Esq.; and "Going-to-the-Sun Highway," by Jean Ewen and A. V. Emery, Esquires.

Irrigation and Power Divisions (Joint Session)

July 12, 1934, "Central Valley Project of California," by Edward Hyatt, M. Am. Soc. C. E.; "Flow of Water Through Quarter-Turn Draft Tubes," by C. A. Mockmore, M. Am. Soc. C. E.; "The Bonneville Power House and Generating Equipment," by B. E. Torpen, M. Am. Soc. C. E.; and "The Silt Problem," by J. C. Stevens, M. Am. Soc. C. E.

Sanitary Engineering Division

January 18, 1934, "On the Flow of Water Through Sand," by Gordon M. Fair, M. Am. Soc. C. E.; "Underground Corrosion," by K. H. Logan, Esq.; Report of Committee on Filtering Materials, by W. E. Stanley, Assoc. M. Am. Soc. C. E.; Report of Committee on Salvage of Sewage, by A. M. Rawn, M. Am. Soc. C. E.; and Report of Committee on Water Supply Engineering, by Thomas H. Wiggin, M. Am. Soc. C. E.

July 12, 1934, "Sanitary Engineering Experiences with a Large Public Water Supply Enterprise in the West," by Joseph D. De Costa, Assoc. M. Am. Soc. C. E.; "Experiences with Certain Troublesome Constituents of Pacific Coast Waters," by Jeptha A. Wade, M. Am. Soc. C. E.; "Municipal Refuse Problems and Procedures on the Pacific Coast," by Chester G. Gillespie, M. Am. Soc. C. E.; and Progress Report of Committee on Salvage of Sewage, by A. M. Rawn, M. Am. Soc. C. E.

Structural Division

January 18, 1934, Report of the Committee on Bridge Floors, by Shortridge Hardesty, M. Am. Soc. C. E.; Report of Committee on Concentrated Loads on Concrete Slabs, by C. T. Morris, M. Am. Soc. C. E.; and "Reconstruction of the Smithfield Street Bridge, Pittsburgh, with Aluminum Floor," by J. P. Growdon, M. Am. Soc. C. E.

Waterways Division

January 18, 1934, "Field Verification of Hydraulic Laboratory Results," by Herbert D. Vogel, Assoc. M. Am. Soc. C. E.; and "Littoral Drift," by J. J. Hennebique, M. Am. Soc. C. E.

MEMBERSHIP OF TECHNICAL DIVISIONS

City Planning	1 512
Construction	2 634
Engineering-Economics and Finance	503
Highway	2 185
Irrigation	958
Power	862
Sanitary Engineering	1 634
Structural	2 746
Surveying and Mapping	960
Waterways	888
Total	14 882

STUDENT CHAPTERS

There are at present 109 Student Chapters. The Ole Miss (University of Mississippi) Student Chapter was reinstated and the University of Georgia Student Chapter was disbanded during the year.

The reports of the Secretary and Treasurer are appended.

By order of the Board of Direction,

George T. Seabury,

Secretary.

January 14, 1935.

BALANCE SHE

ASSETS

Cash:			
In banks and on hand..................	$26 658.32		
On deposit with U. S. Post Office........	200.00	$26 858.32	
Marketable securities, at cost, and accrued interest ($25 881.67 at market quotations and accrued interest)............		29 384.91	
Accounts Receivable:			
Members	66 948.12		
Non-members	2 147.91		
	69 096.03		
Less, allowance for doubtful accounts	55 000.00	14 096.03	
Inventory of publications on hand, at cost or less......		14 717.46	
Prepaid insurance premiums...........................		220.30	
		85 277.02	
Real Estate:			
Interest in real estate and other assets of United Engineering Society, exclusive of trust funds, at book amount	493 352.60		
218–220 West 57th Street, New York, N. Y., at book amount, less depreciation	597 638.84	1 090 991.44	
Furniture and office equipment, less reserve for depreciation		6 122.06	
Library (at book amount):			
Cash expended for books, etc............	22 122.22		
Donations	72 310.83	94 433.05	$1 276 823.
Fund Investments:			
The Fifty-seventh Street Property Fund:			
Marketable securities, at cost ($74 831 at market quotations).............	86 912.99		
Accrued interest	863.24		
Uninvested cash	9 213.72	96 989.95	
The Freeman Fund:			
Marketable securities, at book amounts ($14 765 at market quotations)......	20 235.87		
Uninvested cash	492.85	20 728.72	
J. Waldo Smith Fund:			
Marketable securities, at par value ($20 058 at market quotations)		20 000.00	
Merritt Haviland Smith Memorial Fund...........		1 229.89	
Rudolph Hering Medal Fund..........................		543.03	139 491.5
Assets applicable to unexpended balances of income:			
Cash in banks		6 152.76	
Accrued interest		86.98	6 239.7
			$1 422 554.9

To THE BOARD OF DIRECTION,
AMERICAN SOCIETY OF CIVIL ENGINEERS:

We have examined the accounts of AMERICAN SOCIETY OF CIVIL ENGINEERS
stated therein, and subject to the reasonableness of the amounts at which real esta
financial condition of the Society at that date.

New York, January 10, 1935.

DECEMBER 31, 1934

LIABILITIES AND FUNDS

35 membership dues paid in advance..	$42 100.90		
her member and non-member credits....	4 536.56	$46 637.46	
izes, library and compounded dues fund	23 852.50		
fred Noble Fund....................	16 342.55	40 195.05	
rrent fund surplus, including amount arising from revaluation of real estate..........................		1 189 991.06	$1 276 823.57
nds:			
e Fifty-seventh Street Property Fund.............		96 989.95	
e Freeman Fund..........................		20 728.72	
Waldo Smith Fund..................		20 000.00	
erritt Haviland Smith Memorial Fund............		1 229.89	
dolph Hering Medal Fund........................		543.03	139 491.59
nexpended balances of income:			
Committee on Stresses in Railroad Track...........		60.28	
Freeman Fund income and expenses................		1 145.99	
Special Committee on Earths and Foundations......		1 402.46	
Special Committee on Concrete and Reinforced Concrete Arches..............................		525.12	
Power Division		2 367.64	
City Planning Division..........................		578.85	
Surveying and Mapping Division..................		90.00	
Alfred Noble Fund, not released..................		55.00	
J. Waldo Smith Fund income and expenses..........		14.40	6 239.74
			$1 422 554.90

t December 31, 1934, and, upon the basis of carrying securities at the amounts as nd library are included, in our opinion, the above balance sheet sets forth the

LYBRAND, ROSS BROS. & MONTGOMERY,
Accountants and Auditors.

REPORT OF SECRETARY FOR THE

TO THE BOARD OF DIRECTION OF THE

GENTLEMEN:—I have the honor to present a statement of Receipts and
There is also appended a general Balance Sheet showing the condition of the

RECEIPTS

Cash on hand, January 1, 1934			$46 225.82*
Entrance Fees		$11 880.00	
Current Dues		161 146.86	
Past Dues		14 250.94	
Advance Dues		42 100.90	
Sale of Publications		9 616.91	
Binding for Members		14 832.00	
Badges		3 430.00	
Certificates of Membership		313.00	
Annual Meeting		2 713.25	
Interest on Investments		895.00	
Postage		462.65	
Advertising		22 366.58	
Miscellaneous		606.25	
Income from 57th Street Property:			
Credited to General Receipts	$45 000		
Credited to 57th Street Property Fund	7 500	52 500.00	
From Engineering Foundation in Credit to:			
Special Committee on Earths and Foundations	$1 500		
Special Committee on Concrete and Reinforced Concrete Arches	1 500	3 000.00	
The 57th Street Property Fund:			
Sale of Securities		7 056.80	
Interest on Securities		2 646.85	
Interest Accrued		176.04	
The Freeman Fund:			
Income		1 172.24	
Principal (Sale of Securities)		1 283.61	
Interest Accrued		9.48	
The Alfred Noble Fund:			
Interest on not Released Funds		2.62	
Estate of J. Waldo Smith:			
Bequest		20 000.00	
Interest		252.94	
Interest Accrued		58.68	
City Planning Division:			
Interest		16.40	
Power Division:			
Interest		66.79	
Rudolph Hering Medal Fund:			
Interest		12.00	
Merritt H. Smith Memorial:			
Interest		34.73	
Construction Code Authority:			
Repayment of Loan		500.00	373 403.52
			$419 629.34

* For itemization see p. 20.

YEAR ENDING DECEMBER 31, 1934
AMERICAN SOCIETY OF CIVIL ENGINEERS,
Disbursements for the fiscal year of the Society, ending December 31, 1934.
affairs of the Society.

Respectfully submitted,
GEORGE T. SEABURY, *Secretary.*

DISBURSEMENTS

Salaries of Officers	$18 906.20
Retirement Allowances	2 086.80
Clerical Help	63 252.85
Traveling Allowance of Officers	15 774.36
Rent	10 191.96
Telephone	2 040.44
General Publications	5 798.33
General Printing	2 240.53
Postage	5 305.10
Binding	5 960.68
Badges	2 485.84
Certificates	334.81
Annual Prizes	898.61
Office Supplies	3 655.15
Furniture and Office Equipment	1 477.02
Current Business	5 202.03
Interest on Mortgage	450.00
Insurance	245.22
Reading Room	464.25
Miscellaneous	899.25
Employment Service	4 125.86
Library	8 330.00
American Standards Association	500.00
Local Sections	9 772.15
Technical Publications	108 528.04
Meetings	7 368.99
Technical Divisions	2 905.90
Technical Committees	3 933.93
Administrative Committees	792.68
Professional Committees	3 502.27
American Engineering Council	6 000.00
Construction League	856.57
Payment of Mortgage	18 000.00
The 57th St. Property Fund:	
Purchase of Securities	30 956.25
Interest Accrued	176.04
The Freeman Fund:	
Principal (Purchase of Securities)	1 033.75
Interest Accrued	9.48
Estate of J. Waldo Smith:	
Purchase of Securities	20 318.75
Interest Accrued	58.68
Construction Code Authority, Loan	500 00
	$375 338.77
Cash on hand December 31, 1934	44 290.57*
	$419 629.34

Itemized Statement of Cash on Hand January 1, 1934

Society Funds in Chase National Bank 23d Street.	$11 580.74	
Society Funds in Chase National Bank 41st Street.	500.00	
Petty Cash (in hands of Secretary)	5 000.00	$17 080.74
The 57th St. Property Fund		22 966.32
Power Division		2 300.85
City Planning Division		562.45
Surveying and Mapping Division		90.00
Special Committee on Stresses in Railroad Track		60.28
Special Committee on Earths and Foundations		1 141.00
The Freeman Fund:		
Principal		242.99
The Alfred Noble Fund:		
Not Released		55.00
The Merritt H. Smith Memorial Fund:		
Principal	$1 076.50	
Income and Expense	118.66	1 195.16
The Rudolph Hering Medal Fund:		
Principal	$455.97	
Income and Expense	75.06	531.03
		$46 225.82

Itemized Statement of Cash on Hand December 31, 1934

Society Funds in Chase National Bank 23d St.	$21 030.30	
Society Funds in Chase National Bank 41st St.	500.00	
Society Funds in Railroad Cooperative Bank	62.21	
Petty Cash (in Hands of Secretary)	5 000.00	$26 592.51
The 57th St. Property Fund		9 213.72
Power Division		2 367.64
City Planning Division		578 85
Surveying and Mapping Division		90.00
Special Committee on Stresses in Railroad Track		60.28
Special Committee on Earths and Foundations		1 402.46
Special Committee on Concrete and Reinforced Concrete Arches		525 12
The Freeman Fund:		
Principal	$492.85	
Income and Expense Account	1 139.22	1 632.07
The Alfred Noble Fund:		
Not Released		55.00
The Merritt H. Smith Memorial Fund:		
Principal	$1 076.50	
Income and Expense Account	153.39	1 229.89
The Rudolph Hering Medal Fund:		
Principal	$455.97	
Income and Expense Account	87.06	543.03
		$44 290.57

REPORT OF THE TREASURER OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS FOR THE YEAR ENDING DECEMBER 31, 1934

In compliance with the provisions of the Constitution, I have the honor to present the following report:

Cash on hand January 1, 1934		$46 225.82
Receipts		
From current sources, January 1 to December 31, 1934	$300 008.52	
Rent from 57th Street Property	52 500.00	
Bequest from J. Waldo Smith	20 000.00	
Interest on Investments	895.00	373 403.52
Disbursements		
Payment of Bills by audited vouchers, January 1 to December 31, 1934	$357 338.77	
Final payment on Mortgage	18 000.00	
Cash on hand December 31, 1934	44 290.57	
	$419 629.34	$419 629.34

Respectfully submitted

OTIS E. HOVEY,

Treasurer.